Radical Review Notes and Practice

**Definition of Square Root:** The square root of a number $n$ is the number that gives $n$ when multiplied by itself. Because $a \cdot a = n$.

- **radical** – An expression written with a radical symbol $\sqrt{}$
- **radicand** – the number or expression written inside a radical symbol. For $\sqrt{3}$, the radicand is 3.

**Example 1:** Determine the square roots of the following numbers.

1) $\sqrt{9} =$ _______ because ______$ \cdot $ ______ = 9
2) $\sqrt{64} =$ _______ because ______$ \cdot $ ______ = 64
3) $\sqrt{169} =$ _______ because ______$ \cdot $ ______ = 169
4) $\sqrt{n^2} =$ _______ because ______$ \cdot $ ______ = $n^2$

**Rules for Simplifying Radical Expressions**
1. No radicand can have perfect square factors other than 1.
2. No radicands can contain fractions.
3. No radicals can appear in the denominator of a fraction.

**Product Property of Square Roots**
For any positive real numbers $a$ and $b$, $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$.

**Example 2:** Simplify the following. Show your work.

\[
\sqrt{18} = \sqrt{3 \cdot 3 \cdot 2} \quad \text{Prime factorization of 18}
\]
\[
= \sqrt{3^2 \cdot 2} \quad \text{Product property of square roots}
\]
\[
= 3\sqrt{2}
\]

1) $\sqrt{140} =$
2) $\sqrt{72x^3y^2} =$
3) $\sqrt{5} \cdot \sqrt{35} =$
**Quotient Property of Square Roots**

For any positive real numbers $a$ and $b$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

**Example 3:** Simplify the following. Show your work.

$$\frac{\sqrt{56}}{\sqrt{7}} = \sqrt{\frac{56}{7}}$$

Quotient property of square roots

$$= \sqrt{8}$$

$$= \sqrt{4} \cdot \sqrt{2}$$

$$= 2\sqrt{2}$$

1) $\sqrt{\frac{34}{25}}$

2) $\sqrt{\frac{84}{4}}$

3) $\sqrt{\frac{121}{9}}$

**Rationalizing the Denominator**

The process of eliminating a radical from the denominator of a fraction by multiplying both the numerator and the denominator by an appropriate radical.

**Example 4:** Rationalize the denominator and simplify. Show your work.

$$\frac{\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{5} \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

Note that $\frac{\sqrt{3}}{\sqrt{3}} = 1$

$$= \frac{\sqrt{15}}{3}$$

1) $\frac{\sqrt{7}}{\sqrt{12}}$

2) $\sqrt{\frac{11}{7}}$

3) $\sqrt{\frac{3}{5}}$

4) $\frac{3\sqrt{5}}{\sqrt{3}}$

**PEANUTS®**

"You're lucky, do you know that, bird? You're lucky because you don't have to study math!"

"You don't have to know about rationalizing the denominator and dumb things like that!"

"You're really lucky!"

$\frac{7\sqrt{2}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{7\sqrt{23}}{6}$

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For all real numbers \(a\) and \(b\) where \(a \geq 0, b \geq 0\):

\[
\left(\sqrt{a}\right)^2 = a \\
\sqrt{a^2} = a
\]

**Example 5:** Simplify. Show your work.

1) \(\left(\sqrt{7}\right)^2\)

2) \(\sqrt{(2x)^2}\)

3) \(\left(2\sqrt{3}\right)^2\)

4) \(\sqrt{18}^2\)

5) \(\sqrt{(3ab)^2}\)

6) \(\left(5\sqrt{25}\right)^2\)

**Example 6:** Simplify. Show your work.

**Addition and Subtraction Properties of Radicals**

For any positive real numbers \(a\), \(b\) and \(c\) where \(b \geq 0\):

\[
a\sqrt{b} + c\sqrt{b} = (a + c)\sqrt{b} \\
a\sqrt{b} - c\sqrt{b} = (a - c)\sqrt{b}
\]

\[
6\sqrt{7} + 5\sqrt{7} - 3\sqrt{7} = (6 + 5 - 3)\sqrt{7} \\
= 8\sqrt{7}
\]

1) \(5\sqrt{6} + 3\sqrt{7} + 4\sqrt{7} - 2\sqrt{6}\)

2) \(4\sqrt{27} + 5\sqrt{12} + 8\sqrt{75}\)