Introduction to Polynomials
Postal Service

ACTIVITY 14
continued

ACTIVITY 14 PRACTICE
Write your answers on notebook paper.
Show your work.

Lesson 14-1

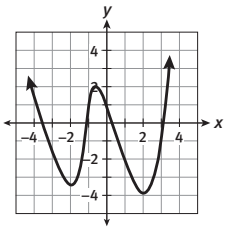
1. The volume of a rectangular box is given by the expression $V = (120 - 6w)w^2$, where w is measured in inches.
   - **a.** What is a reasonable domain for the function in this situation? Express the domain as an inequality, in interval notation, and in set notation.
   - **b.** Sketch a graph of the function over the domain that you found. Include the scale on each axis.
   - **c.** Use a graphing calculator to find the coordinates of the maximum point of the function.
   - **d.** What is the width of the box, in inches, that produces the maximum volume?
2. A cylindrical can is being designed for a new product. The height of the can plus twice its radius must be 45 cm.
   - **a.** Find an equation that represents the volume of the can, given the radius.
   - **b.** Find the radius that yields the maximum volume.
   - **c.** Find the maximum volume of the can.

Lesson 14-2

3. Sketch the graph of the polynomial function $f(x) = -x^3 + 4x^2 - 4x$.
4. Name any x- or y-intercepts of the function $f(x)$ in Item 3.
5. Name any relative maximum values and relative minimum values of the function $f(x)$ in Item 3.

For Items 6–10, decide if each function is a polynomial. If it is, write the function in standard form, and then state the degree and leading coefficient.

6. $f(x) = 7x^2 - 9x^3 + 3x^7 - 2$
7. $f(x) = 2x^3 + x - 5^x + 9$
8. $f(x) = x^4 + x + 5 - \frac{1}{4}x^3$
9. $f(x) = -0.32x^3 + 0.08x^4 + 5^{x-1} - 3$
10. $f(x) = 3x + 5 + \sqrt{x}$
11. Examine the graph below.

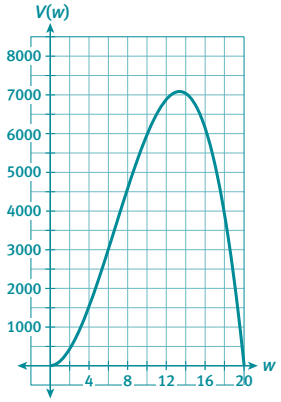


Which of the following statements is NOT true regarding the polynomial whose graph is shown?

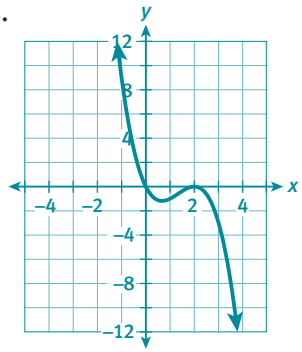
A. The degree of the polynomial is even.
B. The leading coefficient is positive.
C. The function is a second-degree polynomial.
D. As $x \to \pm\infty$, $y \to \infty$.

ACTIVITY PRACTICE

1. **a.** $0 < w < 20$; $(0, 20)$; $\{w | w \in \Re, 0 < w < 20\}$
   b.



   c. (13.333, 7111.111)
   d. about 13.3 inches
2. **a.** $V = -2\pi r^3 + 45\pi r^2$
   b. 15 cm
   c. about 10,603 cm^3
3.



4. x-intercepts: 0 and 2; y-intercept: 0
5. relative maximum value: 0; relative minimum value: −1.185
6. Yes; $f(x) = 3x^7 - 9x^3 + 7x^2 - 2$; seventh degree; 3
7. No.
8. Yes; $f(x) = x^4 - \frac{1}{4}x^3 + x + 5$; fourth degree; 1
9. No.
10. No.
11. C

ACTIVITY 14 Continued

12. As $x \to \pm\infty$, $f(x) \to \infty$.
13. As $x \to \infty$, $f(x) \to -\infty$, and as $x \to -\infty$, $f(x) \to \infty$.
14. Polynomials are continuous functions. Since one side of the graph increases without bound and the other side decreases without bound, the graph must cross the x-axis in at least one place.
15. Check students' work.
16. even
17. neither
18. odd
19. B
20. Check students' work.
21. $(-5, 3)$; Since an even function is symmetric over the y-axis, you can reflect the point $(5, 3)$ over the y-axis to get the point $(-5, 3)$.
22. Sharon is correct that the function is a polynomial function and that it has a positive leading coefficient. However, the function is not an even function because it is not symmetric over the y-axis. She is also incorrect about the degree; since the graph crosses the x-axis four times, it must be at least a fourth-degree polynomial.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

ACTIVITY 14
continued

Introduction to Polynomials
Postal Service

For Items 12 and 13, describe the end behavior of each function using arrow notation.

12. $f(x) = x^6 - 2x^3 + 3x^2 + 2$
13. $f(x) = -x^3 + 7x^2 - 11$
14. Use the concept of end behavior to explain why a third-degree polynomial function must have at least one x-intercept.
15. Sketch a graph of any third-degree polynomial function that has exactly one x-intercept, a relative minimum at $(-2, 1)$, and a relative maximum at $(4, 3)$.

Lesson 14-3

For Items 16–28, determine whether each function is even, odd, or neither.

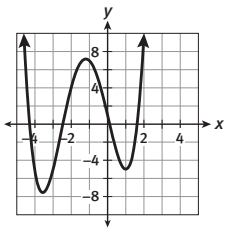
16. $f(x) = 10 + 3x^2$
17. $f(x) = -x^3 + 2x + 5$
18. $f(x) = 6x^5 - 4x$
19. When graphed, which of the following polynomial functions is symmetric about the origin?
    A. $f(x) = -x^3 + 2x + 5$
    B. $f(x) = x^3 + 8x$
    C. $f(x) = -7x^2 + 5$
    D. $f(x) = 5x^3 + 3x^2 - 7x + 1$
20. Sketch a graph of an even function whose degree is greater than 2.
21. If $f(x)$ is an even function and passes through the point $(5, 3)$, what other point must lie on the graph of the function? Explain your reasoning.

MATHEMATICAL PRACTICES

Construct Viable Arguments and Critique the Reasoning of Others

22. Sharon described the function graphed below as follows:
    - It is a polynomial function.
    - It is an even function.
    - It has a positive leading coefficient.
    - The degree n could be any even number greater than or equal to 2.

    Critique Sharon's description. If you disagree with any of her statements, provide specific reasons as to why.

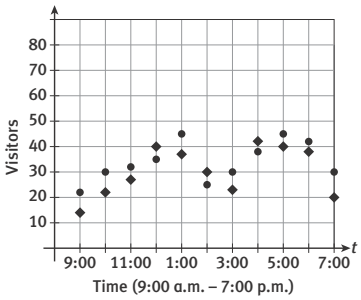


ACTIVITY 15 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 15-1

1. The graph below shows the number of visitors at a public library one day between the hours of 9:00 a.m. and 7:00 p.m. The round dots represent $A(t)$, the number of adult visitors, and the diamonds represent $C(t)$, the number of children and teenage visitors. Graph $V(t)$, the total number of visitors, and explain how you used the graph to find the values of $V(t)$.



2. Examine the functions graphed in Item 1. Which of the statements is true over the given domain of the functions?
A. $A(t) > C(t)$
B. $C(t) > A(t)$
C. $A(t) - C(t) > 0$
D. $V(t) > C(t)$

3. The polynomial expressions $5x + 7$, $3x^2 + 9$, and $3x^2 - 2x$ represent the lengths of the sides of a triangle for all whole-number values of $x > 1$. Write an expression for the perimeter of the triangle.

4. In Item 3, what kind of expression is the perimeter expression?

Lesson 15-2

5. An open box will be made by cutting four squares of equal size from the corners of a 10-inch-by-12-inch rectangular piece of cardboard and then folding up the sides. The expression $V(x) = x(10 - 2x)(12 - 2x)$ can be used to represent the volume of the box. Write this expression as a polynomial in standard form.

6. Write an expression for the volume of a box that is constructed in the same way as in Item 5, but from a rectangular piece of cardboard that measures 8 inches by 14 inches. Write your expression in factored form, and then as a polynomial in standard form.

7. Write an expression to represent the combined volume of the two boxes described in Items 5 and 6.

For Items 8–13, find each sum or difference.

8. $(3x - 4) + (5x + 1)$

9. $(x^2 - 6x + 5) - (2x^2 + x + 1)$

10. $(4x^2 - 12x + 9) + (3x - 11)$

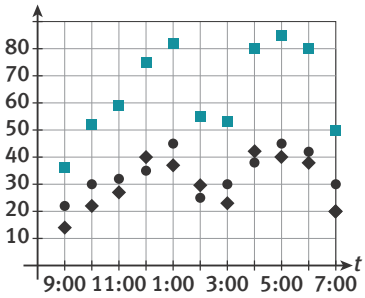
11. $(6x^2 - 13x + 4) - (8x^2 - 7x + 25)$

12. $(4x^3 + 14) + (5x^2 + x)$

13. $(2x^2 - x + 1) - (x^2 + 5x + 9)$

ACTIVITY PRACTICE

1.



Each value of $V(t)$ is the sum of the values for $A(t)$ and $C(t)$ at the given t.

2. D

3. $P(x) = 6x^2 + 3x + 16$

4. a polynomial

5. $V(x) = 4x^3 - 44x^2 + 120x$

6. $V(x) = x(8 - 2x)(14 - 2x)$;
$V(x) = 4x^3 - 44x^2 + 112x$

7. $V(x) = 8x^3 - 88x^2 + 232x$

8. $8x - 3$

9. $-x^2 - 7x + 4$

10. $4x^2 - 9x - 2$

11. $-2x^2 - 6x - 21$

12. $4x^3 + 5x^2 + x + 14$

13. $x^2 - 6x - 8$

ACTIVITY 15 Continued

14. $20x^4 + 15x^3 - 45x^2$

15. $4x^2 - 20x + 25$

16. $x^6 + 2x^3y^3 + y^6$

17. $3x^4 - 2x^3 - 14x^2 - 3x - 14$

18. $2x^4 - 15x^3 + 28x^2 - 9x + 18$

19. A

20. $x - 7 + \frac{11}{x+1}$

21. $5x^2 - x - 2 + \frac{16x+2}{x^2+3x+1}$

22. $2x^2 + x + 6 + \frac{5}{x-2}$

23. $x - 4 + \frac{20}{x+4}$

24. $3x^2 + 2x + 20 + \frac{58}{x-4}$

25. $2x^2 - 10x + 15 - \frac{41}{x+3}$

26. a. Yes; check students' work. Students' examples should show the highest-order terms summing to zero.

b. No; because you are combining like terms, there is no way for the degree to increase.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

For Items 14–18, find each product. Write your answer as a polynomial in standard form.

14. $5x^2(4x^2 + 3x - 9)$

15. $(2x - 5)^2$

16. $(x^3 + y^3)^2$

17. $(x + 2)(3x^3 - 8x^2 + 2x - 7)$

18. $(x - 3)(2x^3 - 9x^2 + x - 6)$

Lesson 15-3

19. Which of the following quotients CANNOT be found using synthetic division?

A. $\frac{x^3 + 4x^2 + 5}{x^2 + 1}$

B. $\frac{-x^2 - x^2 + 1}{x - 1}$

C. $\frac{x^5 + 10}{x + 50}$

D. $\frac{2x^3}{x+1}$

For Items 20–22, find each quotient using long division.

20. $\frac{x^2 - 6x + 4}{x+1}$

21. $(5x^4 + 14x^3 + 9x) \div (x^2 + 3x + 1)$

22. $(2x^3 - 3x^2 + 4x - 7) \div (x - 2)$

For Items 23–25, find each quotient using synthetic division.

23. $(x^2 + 4) \div (x + 4)$

24. $\frac{3x^3 - 10x^2 + 12x - 22}{x - 4}$

25. $(2x^3 - 4x^2 - 15x + 4) \div (x + 3)$

MATHEMATICAL PRACTICES

Reason Abstractly and Quantitatively

26. Before answering parts a and b, review them carefully to ensure you understand all the terminology and what is being asked.

a. When adding two polynomials, is it possible for the degree of the sum to be less than the degree of either of the polynomials being added (the addends)? If so, give an example to support your answer. If not, explain your reasoning.

b. Is it possible for the degree of the sum to be greater than the degree of either of the addends? If so, give an example to support your answer. If not, explain your reasoning.

ACTIVITY 16 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 16-1

1. Which of the following would you use to find the number of different combinations of six-person nominating committees that could be chosen from a class of 25 students?

   A. ${}_6C_{25} = \frac{6!}{25!(25-6!)}$

   B. ${}_{25}C_6 = \frac{25!}{25!(25-6!)}$

   C. ${}_{25}C_6 = \frac{25!}{6!(25-6!)}$

   D. ${}_6C_{25} = \frac{6!}{6!(25-6!)}$

2. Simplify: $\frac{9\times8\times7\times6\times5\times4\times3\times2\times1}{(6\times5\times4\times3\times2\times1)(3\times2\times1)}$

3. Write the expression in Item 2 in ${}_nC_r$ notation.

4. Find the number of different combinations of four-person nominating committees that could be chosen from a class of 25 students.

5. Write the numbers that will fill in the eighth row of Pascal's triangle.

6. In which row of Pascal's triangle would you find the coefficients for the terms in the expansion of $(a+b)^{14}$?

7. Which of the following has the same value as $\binom{12}{7}$?

   A. ${}_{12}C_7$

   B. ${}_{12}C_5$

   C. $\binom{12}{5}$

   D. all of the above

8. Use what you have learned about the patterns in Pascal's triangle to expand $(a+b)^8$.

9. Manuela started expanding $(x+y)^9$. So far, she has written:

   $x^9 + 9x^8y + 36x^7y^2 + 84x^6y^3 + 126x^5y^4 + 126x^4y^5$

   Manuela explained to Karen that since both coefficients in the binomial are 1, the coefficients of the terms will start repeating, only backwards. Use Manuela's strategy to complete the expansion.

ACTIVITY 16 Continued

ACTIVITY PRACTICE

1. C
2. 84
3. ${}_9C_6$
4. 12,650 combinations
5. 1 7 21 35 21 7 1
6. 15th row
7. D
8. $a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8$
9. $84x^3y^6 + 36x^2y^7 + 9xy^8 + y^9$

10. $\sum_{k=0}^{9}\binom{9}{k}a^{9-k}b^{k}$
11. $\sum_{k=0}^{7}\binom{7}{k}(2x)^{7-k}(-3)^{k}$
12. 640
13. D
14. $24x^5$
15. $-4320x^3$
16. $x^4 + 20x^3 + 150x^2 + 500x + 625$
17. $1024a^5 + 1280a^4b + 640a^3b^2 + 160a^2b^3 + 20ab^4 + b^5$
18. $x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$
19. $8x^3 + 12x^2y + 6xy^2 + y^3$
20. **a.** $x^{10} + 5x^8 + 10x^6 + 10x^4 + 5x^2 + 1$; This expansion disproves the statement since the powers of x decrease by 2, not 1.
    b. Sample answer: In the expansion of every linear binomial, $(x + a)^n$, the powers of x decrease by 1 from left to right when written as a polynomial in standard form.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

Lesson 16-2

10. Write $(a + b)^9$ using summation notation.
11. Write $(2x - 3)^7$ using summation notation.
12. Find the coefficient of the fourth term in the expansion of $(x + 4)^5$.
13. Which of the following is the coefficient of the third term in the expansion of $(x - 2)^7$?
    A. -84
    B. -21
    C. 21
    D. 84
14. Find the second term in the expansion of $(x + 4)^6$.
15. Find the fourth term in the expansion of $(3x - 2)^6$.
16. Use the Binomial Theorem to write the binomial expansion of $(x + 5)^4$.
17. Use the Binomial Theorem to write the binomial expansion of $(4a + b)^5$.
18. Use the Binomial Theorem to write the binomial expansion of $(x - 3)^5$.
19. Use the Binomial Theorem to write the binomial expansion of $(2x + y)^3$.

MATHEMATICAL PRACTICES

Make Sense of Problems and Persevere in Solving Them

20. Consider the statement below.

    In the expansion of every binomial, the powers of x decrease by 1 from left to right when written as a polynomial in standard form.

    a. Expand the binomial $(x^2 + 1)^5$ and state whether the expansion supports or disproves the statement above and why.
    b. If the expansion disproves the statement, modify it so that it becomes a true statement.

ACTIVITY 17 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 17-1

1. State the common factor of the terms in the polynomial $5x^3 + 30x^2 - 10x$. Then factor the polynomial.
2. Which of the following is one of the factors of the polynomial $15x^2 - x - 2$?
   A. $x - 2$
   B. $5x - 2$
   C. $5x + 1$
   D. $3x - 1$
3. Factor each polynomial.
   a. $6x^2 + 7x - 5$
   b. $14x^2 + 25x + 6$
4. Factor by grouping.
   a. $8x^3 - 64x^2 + x - 8$
   b. $12x^4 + 2x^3 - 30x - 5$
5. Factor each difference or sum of cubes.
   a. $125x^3 + 216$
   b. $x^6 - 27$
6. Use the formulas for factoring quadratic binomials and trinomials to factor each expression.
   a. $x^4 - 14x^2 + 33$
   b. $81x^4 - 625$
   c. $x^4 + 17x^2 + 60$
   d. $x^6 - 100$

Lesson 17-2

7. Which theorem states that a polynomial of degree n has exactly n linear factors, counting multiple factors?
   A. Binomial Theorem
   B. Quadratic Formula
   C. Fundamental Theorem of Algebra
   D. Complex Conjugate Root Theorem
8. Find the zeros of the functions by factoring and using the Zero Product Property. Identify any multiple zeros.
   a. $f(x) = 2x^4 + 18x^2$
   b. $g(x) = 3x^3 - 3$
   c. $h(x) = 5x^3 - 6x^2 - 45x + 54$
   d. $h(x) = 3x^4 - 36x^3 + 108x^2$
9. The table of values shows coordinate pairs on the graph of $f(x)$. Which of the following could be $f(x)$?
   A. $x(x + 1)(x - 1)$
   B. $(x - 1)(x + 1)(x - 3)$
   C. $(x + 1)^2(x + 3)$
   D. $(x + 1)(x - 2)^2$

x	$f(x)$
−1	0
0	3
1	0
2	−3

10. Write a polynomial function of nth degree that has the given zeros.
    a. $n = 3$; $x = 1$, $x = 6$, $x = -6$
    b. $n = 4$; $x = -3$, $x = 3$, $x = 0$, $x = 4$

ACTIVITY 17 Continued

ACTIVITY PRACTICE

1. $5x$; $5x(x^2 + 6x - 2)$
2. B
3. **a.** $(3x + 5)(2x - 1)$
   b. $(7x + 2)(2x + 3)$
4. **a.** $(x - 8)(8x^2 + 1)$
   b. $(6x + 1)(2x^3 - 5)$
5. **a.** $(5x + 6)(25x^2 - 30x + 36)$
   b. $(x^2 - 3)(x^4 + 3x^2 + 9)$
6. **a.** $(x^2 - 3)(x^2 - 11)$
   b. $(3x + 5)(3x - 5)(9x^2 + 25)$
   c. $(x^2 + 5)(x^2 + 12)$
   d. $(x^3 + 10)(x^3 - 10)$
7. C
8. **a.** $x = \pm 3i$, $x = 0$ (double)
   b. $x = 1$, $x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$
   c. $x = \pm 3$, $x = \frac{6}{5}$
   d. $x = 0$ (double), $x = 6$ (double)
9. B
10. **a.** $f(x) = x^3 - x^2 - 36x + 36$
    b. $f(x) = x^4 - 4x^3 - 9x^2 + 36x$

ACTIVITY 17 Continued

11. B

12. **a.** $f(x) = x^3 + 2x^2 - 25x - 50$
b. $f(x) = x^4 + 16x^2 - 225$
c. $f(x) = x^3 + x + 10$

13. **a.** $n = 4$
b. $n = 5$
c. $n = 4$

14. A

15. Sample answer: The polynomial in II is a fifth-degree polynomial; if you multiply the factors in III together, the constant term will not equal 48.

16. $f(x) = (x - 4)^2(x - i)(x + i)$;
$f(x) = x^4 - 8x^3 + 17x^2 - 8x + 16$

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

ACTIVITY 17
continued

Factors of Polynomials
How Many Roots?

11. Which of the following polynomial functions has multiple roots at $x = 0$?
A. $f(x) = x^2 - x$
B. $f(x) = x^3 - x^2$
C. $f(x) = x^3 - x$
D. all of the above

12. Write a polynomial function of nth degree that has the given real or complex roots.
a. $n = 3; x = -2, x = 5, x = -5$
b. $n = 4; x = -3, x = 3, x = 5i$
c. $n = 3; x = -2, x = 1 + 2i$

13. Give the degree of the polynomial function with the given real or complex roots.
a. $x = -7, x = 1, x = 4i$
b. $x = -2, x = 2, x = 0, x = 4 + i$
c. $x = 2i, x = 1 - 3i$

14. Which of the following could be the factored form of the polynomial function $f(x) = x^4 + \ldots + 48$?
I. $f(x) = (x + 1)(x + 3)(x + 4i)(x - 4i)$
II. $f(x) = (x + 2)^2(x - 1)(x + 4)(x - 6)$
III. $f(x) = (x + 3)(x - 8)(x + 2i)(x - 2i)$
A. I only
B. I and II only
C. II only
D. I, II, and III

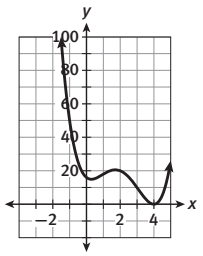
15. Explain your reason(s) for eliminating each of the polynomials you did not choose in Item 14.

MATHEMATICAL PRACTICES
Use Appropriate Tools Strategically

16. Use the information below to write a polynomial function, first in factored form and then in standard form.

Fact: The graph only touches the x-axis at a double zero; it does not cross through the axis.

Clue: One of the factors of the polynomial is $(x + i)$.



ACTIVITY 18 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 18-1

For Items 1–8, match each equation or description to one of the graphs below.

_____ **1.** an even function with no real roots and a positive leading coefficient

_____ **2.** an even function with three real roots and a negative leading coefficient.

_____ **3.** an odd function with one real root and a negative leading coefficient.

_____ **4.** $f(x) = -ax^3 + b$

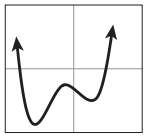
_____ **5.** $g(x) = ax^3 + \ldots + d$

_____ **6.** $h(x) = ax^4 + \ldots - e$

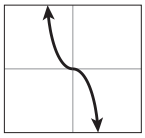
_____ **7.** $p(x) = ax^5 + \ldots - f$

_____ **8.** $p(x) = -ax^5 + \ldots - f$

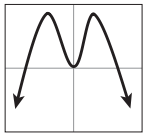
A.



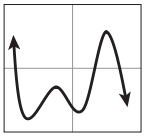
B.



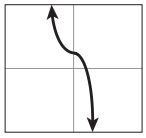
C.



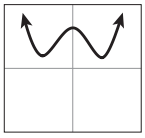
D.



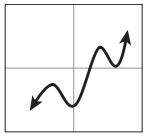
E.



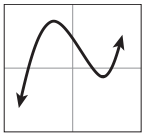
F.



G.



H.



For Items 9–11, use what you know about end behavior and zeros to graph each function.

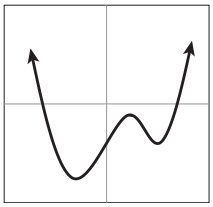
9. $f(x) = x^4 + 2x^3 - 43x^2 - 44x + 84$
$= (x - 1)(x - 6)(x + 2)(x + 7)$

10. $y = x^5 - 14x^4 + 37x^3 + 260x^2 - 1552x + 2240$
$= (x - 7)(x + 5)(x - 4)^3$

11. $f(x) = -x^4 + 11x^3 - 21x^2 - 59x + 70$
$= -(x - 1)(x - 5)(x + 2)(x - 7)$

12. Make a general statement about what information is revealed by an unfactored polynomial compared to a factored polynomial.

13. Miguel identified the graph below as a polynomial function of the form $f(x) = ax^4 - bx^2 + c$, where a, b, and c are positive real numbers.



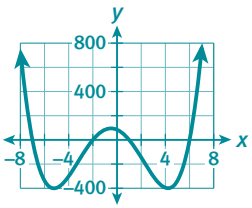
Which reason best describes why Miguel is incorrect?

A. The graph is not a fourth-degree polynomial.
B. The leading coefficient of Miguel's polynomial should be negative.
C. The graph is of an even function, but Miguel's polynomial is not even.
D. The y-intercept is below the x-axis, so Miguel's polynomial should end with $-$ c, not $+$ c.

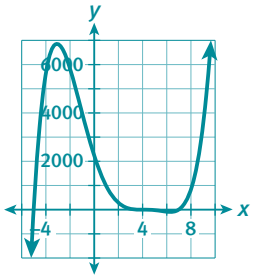
ACTIVITY 18 Continued

ACTIVITY PRACTICE

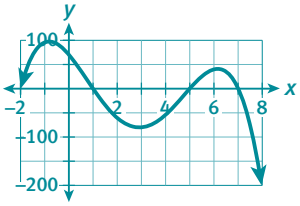
1. F
2. C
3. B
4. E
5. H
6. A
7. G
8. D
9.



10.



11.



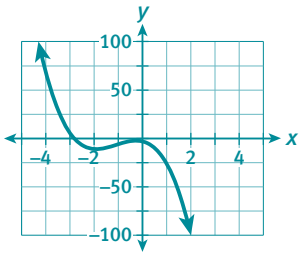
12. An unfactored polynomial reveals information about symmetry, end behavior, the number of relative extrema, and the y-intercept of a graph. Factored polynomials reveal information about x-intercepts of a graph.

13. D

14. a. $\pm\frac{1}{4}, \pm\frac{3}{4}, \pm 1, \pm 3$

b. $\pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$

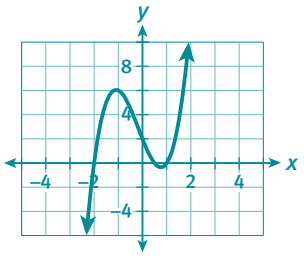
15.



16. a. 2 or 0 possible positive real roots; 1 possible negative real root

b. 2 or 0 possible positive real roots; 2 or 0 possible negative real roots

17.



18. the Complex Conjugate Root Theorem

19. points: $(-4, -180)$, $(-1, -18)$, $(4, 12)$, $(-2, 48)$; zeros: $(1, 0)$, $(2, 0)$

20. points: $(-1.5, -19.5)$, $(-1, -12)$, $(1, 8)$, $(1.5, 29.25)$, $(3, 300)$; zeros: $(-3, 0)$, $(0.5, 0)$

21. B

22. $-4 < x < 2$ and $x > 10$

23. $x < -6$ and $1 < x < 6$

24. $-4 \leq x \leq -2$ and $2 \leq x \leq 4$

25. a. By hand; Sample answer: Since the function is already factored, it will be fairly easy to graph using zeros and end behavior. Also, the scale will be extremely large, so it will take some effort to find a good viewing window in the graphing calculator.

b. Graphing calculator; Sample answer: The function is not factorable, and the possible rational roots include fractions.

c. By hand; Sample answer: There are enough points in the table to get a good idea of what the function looks like. Also, you don't have an equation to enter into the graphing calculator.

ADDITIONAL PRACTICE

If students need more practice on the concepts in this activity, see the Teacher Resources at SpringBoard Digital for additional practice problems.

Lesson 18-2

14. Determine all the possible rational roots of:
a. $f(x) = -4x^3 - 13x^2 - 6x - 3$
b. $g(x) = 2x^4 + 6x^3 - 3x^2 - 11x + 8$

15. Graph $f(x) = -4x^3 - 13x^2 - 6x - 3$.

16. Determine the possible number of positive and negative real roots for:
a. $h(x) = 2x^3 + x^2 - 5x + 2$
b. $p(x) = 2x^4 + 6x^3 - 3x^2 - 11x + 8$

17. Graph $h(x) = 2x^3 + x^2 - 5x + 2$.

18. Descartes' Rule of Signs states that the number of positive real roots of $f(x) = 0$ equals the number of variations in sign of the terms of $f(x)$, or is less than this number by an even integer. What theorem offers a reason as to why the number could be "less than this number by an even integer"?

For Items 19–20, apply the Remainder Theorem to all the possible rational roots of the given polynomial to identify points on the graph or zeros of the polynomial.

19. $p(x) = x^3 - 5x^2 + 8x - 4$

20. $h(x) = 2x^4 + 5x^3 - x^2 + 5x - 3$

21. The graph of $f(x)$ has an x-intercept at $(4, 0)$. Which of the following MUST be true?

I. $f(4) = 0$
II. $x - 4$ is a factor of $f(x)$.
III. $f(x)$ also has an x-intercept at $(-4, 0)$.

A. II only
B. I and II only
C. II and III only
D. I, II, and III

Lesson 18-3

For Items 22–24, solve the polynomial inequality.

22. $(x + 4)(x - 2)(x - 10) > 0$

23. $x^3 - x^2 - 36x + 36 < 0$

24. $-x^4 + 20x^2 - 32 \geq 32$

MATHEMATICAL PRACTICES

Look For and Express Regularity in Repeated Reasoning

25. Some polynomial functions are represented in a variety of forms below. For each representation, describe whether you think it is more efficient to graph the polynomial using a graphing calculator or by hand. Justify your choices.

a. $f(x) = (x + 15)(x + 7)(x - 5)^2(x - 12)$

b. $g(x) = 2x^4 + 6x^3 - 3x^2 - 11x + 7$

c.

x	$f(x)$
−3	−8
−1	1
0	2
1	1
3	−6
4	−2