Proof, Parallel and Perpendicular Lines

Unit Overview
In this unit you will begin the study of an axiomatic system, Geometry. You will investigate the concept of proof and discover the importance of proof in mathematics. You will extend your knowledge of the characteristics of angles and parallel and perpendicular lines and explore practical applications involving angles and lines.

Key Terms
As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary
- compare and contrast
- negate
- contrast
- format
- justify
- confirm
- argument
- interchange

Math Terms
- inductive reasoning
- truth value
- conjecture
- biconditional statement
- deductive reasoning
- postulates
- proof
- midpoint
- theorem
- congruent
- hypothesis
- bisect
- conclusion
- bisector of an angle
- conditional statement
- congruent
- hypothesis
- parallel
- conclusion
- corresponding angles
- counterexample
- perpendicular
- converse
- perpendicular bisector
- inverse
- congruence

ESSENTIAL QUESTIONS
Why are properties, postulates, and theorems important in mathematics?
How are angles and parallel and perpendicular lines used in real-world settings?

EMBEDDED ASSESSMENTS
These assessments, following Activities 3, 5, and 8 will give you an opportunity to demonstrate what you have learned about reasoning, proof, and some basic geometric figures.

Embedded Assessment 1:
Geometric Figures and Basic Reasoning p. 37

Embedded Assessment 2:
Distance, Midpoint, and Angle Measurement p. 61

Embedded Assessment 3:
Angles, Parallel Lines, and Perpendicular Lines p. 99
Write your answers on notebook paper.
Show your work.

1. Solve each equation.
   a. $5x - 2 = 8$
   b. $4x - 3 = 2x + 9$
   c. $6x + 3 = 2x + 8$

2. Graph $y = 3x - 3$ and label the $x$- and $y$-intercepts.

3. Tell the slope of a line that contains the points $(5, -3)$ and $(7, 3)$.

4. Write the equation of a line that has slope $\frac{1}{3}$ and $y$-intercept 4.

5. Write the equation of the line graphed below.

6. Describe a pattern shown in this sequence, and use the pattern to find the next two terms.
   $7, 15, 23, 31, 39, \_\_, \_\_, \ldots$

7. Draw a right triangle and label the hypotenuse and legs.

8. Use a protractor to find the measure of each angle.

   a. 
   b. 
   c. 

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Geometric Figures
What’s My Name?
Lesson 1-1 Basic Geometric Figures

Learning Targets:
- Identify, describe, and name points, lines, line segments, rays, and planes using correct notation.
- Identify and name angles.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Think-Pair-Share, Group Presentation, Interactive Word Wall, Think Aloud, Debriefing, Self Revision/Peer Revision

Below are some types of figures you have seen in earlier mathematics courses. Describe each figure using your own words. If you can recall the mathematical terms that identify the figures, you can use them in your descriptions.

1. Q

2. F G H

3. X Y Z

4. D E

5. A

6. R S T

7. J T P K N
## Naming Geometric Figures

<table>
<thead>
<tr>
<th>Geometric Figure</th>
<th>Naming</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>point</td>
<td>Named with a capital letter</td>
<td>point P</td>
</tr>
<tr>
<td>line</td>
<td>Named using any two points on the line, in any order, with a line symbol drawn above OR Named using a lowercase letter</td>
<td>AB, BA, or line m</td>
</tr>
<tr>
<td>line segment</td>
<td>Named using the two endpoints, in any order, with a segment symbol drawn above</td>
<td>ST or TS</td>
</tr>
<tr>
<td>ray</td>
<td>Named using the endpoint and one other point, with a ray symbol drawn above; the endpoint is always listed first</td>
<td>MN</td>
</tr>
<tr>
<td>plane</td>
<td>Named using any three points in the plane that are not on the same line, in any order OR Named using a capital cursive letter</td>
<td>plane FGH or plane Q</td>
</tr>
</tbody>
</table>

8. Identify each geometric figure. Then give all possible names for the figure.

- a. \(\overrightarrow{QP}\) or \(\overrightarrow{PQ}\)
- b. \(\overrightarrow{NC}\)
- c. Plane \(\overrightarrow{GTM}\)
- d. Segment \(\overrightarrow{DB}\)
Lesson 1-1
Basic Geometric Figures

9. Draw $\overline{FE}$. Explain where the points $F$ and $E$ lie on the ray.

10. Critique the reasoning of others. Caleb says that the figure below can be named $\overline{KJ}$. Jen says the figure can be named $\overline{JL}$. Who is correct? Explain.

Check Your Understanding

11. Is $\overline{SR}$ a possible name for the figure at the right? Explain.

12. Graham draws a point. Describe how he could label and name the point.

There are three different ways to name an angle.

- Use the angle symbol and a number.
- Use the angle symbol and the vertex of the angle.
- Use the angle symbol and three points on the angle. The first point is on one side of the angle, the second point is the vertex, and the third point is on the other side of the angle.

Example A
Give all possible names for the angle.

Use a number: $\angle 1$.

Use the vertex: $\angle B$.

Use three points. The second point should be the vertex. Be sure the first and third points are not on the same side.

$\angle ABC, \angle ABD, \angle CBA, \angle DBA$
Try These A
Give all possible names for each angle.

a. 

b. 

Check Your Understanding

13. Draw a figure that could be named \( \angle LMN \).
14. Is \( \angle FDE \) a possible name for the figure at the right? Explain.
15. How many different line segments does the figure at the right include? Name them.

LESSON 1-1 PRACTICE

16. Identify each geometric figure. Then give all possible names for the figure.

a. 

b. 

c. 

d. 

The diagram below includes \( RU \). Use the figure for Items 17–19.

17. How many different rays does the figure include? Name them.
18. **Reason abstractly.** Explain why \( \angle S \) is not an appropriate name for \( \angle 1 \).
19. Give the other possible appropriate names for \( \angle 2 \).
20. Draw \( LM \). Then draw \( NP \) so that point \( N \) lies on \( LM \).
Learning Targets:
• Describe angles and angle pairs.
• Identify and name parts of circles.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Self Revision/Peer Revision, Discussion Groups, Create Representations

As you share your ideas, be sure to use mathematical terms and academic vocabulary precisely. Make notes to help you remember the meaning of new words and how they are used to describe mathematical ideas.

1. Draw four angles with different characteristics. Describe each angle. Name the angles using numbers and letters.

2. Compare and contrast each pair of angles.
   a.  
   b.  
   c.  
   d.  

Academic Vocabulary

Activity 1 • Geometric Figures
Recall that the sum of the measures of complementary angles is \(90^\circ\) and the sum of the measures of supplementary angles is \(180^\circ\).

3. a. The figure below shows two intersecting lines. Name two angles that are supplementary to \(\angle 4\).

b. **Reason quantitatively.** Explain why the angles you named in part a must have the same measure.

![Diagram of intersecting lines with angles labeled 3, 4, 5, and 6.]

4. Complete the chart by naming all the listed angle types in each figure.

```
<table>
<thead>
<tr>
<th>Acute angles</th>
<th>Obtuse angles</th>
<th>Angles with the same measure</th>
<th>Supplementary angles</th>
<th>Complementary angles</th>
</tr>
</thead>
</table>
```

**MATH TIP**

Angles can be classified by their measures.
- An acute angle measures greater than \(0^\circ\) and less than \(90^\circ\).
- A right angle measures \(90^\circ\).
- An obtuse angle measures greater than \(90^\circ\) and less than \(180^\circ\).
- A straight angle measures \(180^\circ\).
Lesson 1-2
More Geometric Figures

A chord of a circle is a segment with both endpoints on the circle.

A diameter is a chord that passes through the center of a circle.

A radius is a segment with one endpoint on the circle and one endpoint at the center of the circle.

5. In the circle below, draw and label each geometric term.
   a. radius $OA$
   b. chord $BA$
   c. diameter $CA$

6. Refer to your drawings in the circle above. What is the geometric term for point $O$?

7. The circles below are concentric, meaning that they have the same center. The center of both circles is point $P$.
   a. Construct viable arguments. Explain why circle $P$ is not an appropriate name for the smaller circle.

   b. Propose an alternate name for the smaller circle that would be appropriate. Justify your choice.

   When you justify a choice, you provide evidence that shows that your choice is correct or reasonable.
LESSON 1-2 PRACTICE

11. Classify each segment in circle $O$. Use all terms that apply.
   a. $AF$
   b. $BO$
   c. $CO$
   d. $DO$
   e. $EO$
   f. $CE$

The figure below includes $PL$ and $JM$. Use the figure for Items 12–13.

12. Name three pairs of supplementary angles.

13. Name two angles that appear to be obtuse.


15. Model with mathematics. Is it possible for a pair of nonadjacent angles to share vertex $A$ and arm $AB$? If it is possible, draw an example. If it is not possible, explain your answer.
ACTIVITY 1 PRACTICE
Write your answers on notebook paper.
Show your work.

Lesson 1-1

1. Which is a correct name for this line?

   \[ \overline{GEM} \]
   \[ A. \overline{G} \quad B. \overline{GM} \quad C. \overline{MG} \quad D. \overline{ME} \]

2. Draw each figure.
   a. point \( L \)
   b. \( \overline{MN} \)
   c. \( \overline{PQ} \)
   d. plane \( \mathcal{R} \)
   e. \( \overline{ST} \)
   f. \( \angle U \)

3. Identify each geometric figure. Then give all possible names for the figure.
   a. \[ \begin{array}{c} B \quad C \quad D \end{array} \]
   b. \[ \begin{array}{c} P \quad Q \quad R \end{array} \]
   c. \[ \begin{array}{c} M \quad N \quad P \end{array} \]
   d. \( \overline{FG} \)
   e. \[ \begin{array}{c} X \quad Y \quad n \end{array} \]

4. Describe all of the acceptable ways to name a plane.
5. What are some acceptable ways to name an angle?
6. Describe all of the acceptable ways to name a line.
7. a. How many rays are in the figure below?
   Name them.
   b. How many different angles are in the figure?
   Name them.

8. Describe the diagram below using correct geometric names.

9. a. Explain why plane \( JKL \) is not an appropriate name for the plane below.
   b. Give three names for the plane that would be appropriate.

10. How can you use the figures in this activity to describe real-world objects and situations? Give examples.
Lesson 1-2

11. In this diagram, \( m\angle SUT = 25^\circ \).

   \[ \begin{array}{c}
   \text{a. Name another angle that measures } 25^\circ. \\
   \text{b. Name a pair of complementary angles.} \\
   \text{c. Name a pair of supplementary angles.}
   \end{array} \]

   Use circle \( Q \) for Items 12–15.

   \[ \begin{array}{c}
   \text{12. Name the radii of circle } Q. \\
   \text{13. Name the diameter(s) in circle } Q. \\
   \text{14. Name the chord(s) in circle } Q. \\
   \text{15. Which statement below must be true about circle } Q? \\
   \text{A. The distance from } U \text{ to } W \text{ is the same as the distance from } R \text{ to } T. \\
   \text{B. The distance from } U \text{ to } W \text{ is the same as the distance from } Q \text{ to } J. \\
   \text{C. The distance from } R \text{ to } T \text{ is half the distance from } Q \text{ to } R. \\
   \text{D. The distance from } R \text{ to } T \text{ is twice the distance from } Q \text{ to } J.
   \end{array} \]

16. Two angles have the same measure. The angles are also supplementary. Are the angles acute, right, or obtuse? How do you know?

17. This diagram includes \( \overline{KN}, \overline{SP}, \text{ and } \overline{MR} \).

   \[ \begin{array}{c}
   \text{a. Name three angles that appear to be acute.} \\
   \text{b. Name three angles that appear to be obtuse.} \\
   \text{c. Name three straight angles.}
   \end{array} \]

18. \( \angle F \) and \( \angle G \) are complementary. The measure of \( \angle F \) is four times the measure of \( \angle G \). What is the measure of each angle?

19. Compare and contrast a chord and a diameter of a circle.

20. a. Lucinda describes the angle below as \( \angle BAC \). Ahmad describes the angle as \( \angle CBD \). State whether each of these names is appropriate for the angle, and explain why or why not.

   \[ \begin{array}{c}
   \text{b. Give another name for the angle, and explain why the name you chose is appropriate.}
   \end{array} \]
Logical Reasoning
Riddle Me This
Lesson 2-1 Inductive Reasoning

Learning Targets:
- Make conjectures by applying inductive reasoning.
- Recognize the limits of inductive reasoning.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Vocabulary Organizer, Think-Pair-Share, Look for a Pattern, Visualization, Discussion Groups, Self Revision/Peer Revision

The ability to recognize patterns is an important aspect of mathematics. But when is an observed pattern actually a real pattern that continues beyond just the observed cases? In this activity, you will explore patterns and check to see if the patterns hold true beyond the observed cases.

Inductive reasoning is the process of observing data, recognizing patterns, and making a generalization. This generalization is a conjecture.

1. Five students attended a party and ate a variety of foods. Something caused some of them to become ill. JT ate a hamburger, pasta salad, and coleslaw. She became ill. Guy ate coleslaw and pasta salad but not a hamburger. He became ill. Dean ate only a hamburger and felt fine. Judy didn't eat anything and also felt fine. Cheryl ate a hamburger and pasta salad but no coleslaw, and she became ill. Use inductive reasoning to make a conjecture about which food probably caused the illness.

2. Reason quantitatively. Use inductive reasoning to make a conjecture about the next two terms in each sequence. Explain the pattern you used to determine the terms.
   a. A, 4, C, 8, E, 12, G, 16, ______, ______
   b. 3, 9, 27, 81, 243, ______, ______
   c. 3, 8, 15, 24, 35, 48, ______, ______
   d. 1, 1, 2, 3, 5, 8, 13, ______, ______

CONNECT TO HISTORY
The sequence of numbers in Item 2d is named after Leonardo of Pisa, who was known as Fibonacci. Fibonacci’s 1202 book Liber Abaci introduced the sequence to Western European mathematics.
3. Use the four circles below. From each of the given points, draw all possible chords. These chords will form a number of nonoverlapping regions in the interior of each circle. For each circle, count the number of these regions. Then enter this number in the appropriate place in the table below.

<table>
<thead>
<tr>
<th>Number of Points on the Circle</th>
<th>Number of Nonoverlapping Regions Formed</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

4. Look for a pattern in the table above.

a. Describe, in words, any patterns you see for the numbers in the column labeled Number of Nonoverlapping Regions Formed.

b. Use the pattern that you described to predict the number of nonoverlapping regions that will be formed if you draw all possible line segments that connect six points on a circle.
Lesson 2-1
Inductive Reasoning

Check Your Understanding

5. Critique the reasoning of others. The diagram shows the first three figures in a pattern. Mario conjectures that the fourth figure in the pattern will be the third figure rotated $90^\circ$ clockwise. Is Mario's conjecture reasonable? Explain.

![Diagram of figures]

6. Anya is training for a 10K race. The table shows the distance she ran during the first 3 weeks of training.

<table>
<thead>
<tr>
<th>Week</th>
<th>Distance per Practice (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
</tr>
</tbody>
</table>

a. Make a conjecture about the distance Anya will run during each practice of the fourth week.

b. How could you test your conjecture?

7. Use the circle on the next page. Draw all possible chords connecting any two of the six points.

a. What is the number of nonoverlapping regions formed by chords connecting the points on this circle?

b. Is the number you obtained above the same number you predicted from the pattern in the table in Item 4b?

c. Describe what you would do to further investigate the pattern in the number of regions formed by chords joining $n$ points on a circle, where $n$ represents any number of points placed on a circle.

MATH TIP
Number the nonoverlapping regions as you count them. That way, you’ll be sure not to skip any or count any more than once.
d. **Make use of structure.** Try to find a new pattern for predicting the number of regions formed by chords joining points on a circle when two, three, four, five, and six points are placed on a circle. Describe the pattern in algebraic terms or in words.
Lesson 2-1
Inductive Reasoning

Check Your Understanding

8. Zack writes a conjecture that is true for the first few terms of a sequence. Is the conjecture necessarily true for all terms of the sequence? Explain.

9. The table below represents the function \( f(x) = 2x + 5 \). Based on the table, Trent conjectures that the value of \( f(x) \) is always positive. Is Trent’s conjecture true? Explain how you know.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

10. Monique tosses a coin three times. She gets heads each time. Based on this pattern, she makes a conjecture that she will always get heads when she tosses the coin. Do you think Monique’s conjecture is reasonable? Why or why not?

LESSON 2-1 PRACTICE

11. Use inductive reasoning to determine the next two terms in each sequence.

a. \( \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \)

b. A, B, D, G, K, \ldots

12. Write the first five terms of two different sequences that have 10 as the second term. Describe the pattern in each of your sequences.

13. Make sense of problems. Generate a sequence using this description: The first term in the sequence is 4, and each other term is three more than twice the previous term.

14. The diagram shows the first three figures in a pattern. Each figure is made of small triangles. How many small triangles will be in the sixth figure of the pattern? Support your answer.

15. Critique the reasoning of others. The first two terms of a number pattern are 2 and 4. Alicia conjectures that the next term will be 6. Mario conjectures that the next term will be 8. Whose conjecture is reasonable? Explain.

MATH TIP
Recall that a sequence is an ordered list of numbers or other items. For example, 2, 4, 6, 8, 10, \ldots is a sequence.
Learning Targets:

- Use deductive reasoning to prove that a conjecture is true.
- Develop geometric and algebraic arguments based on deductive reasoning.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Vocabulary Organizer, Marking the Text, Think-Pair-Share, Look for a Pattern, Use Manipulatives, Discussion Groups, Summarizing, Paraphrasing, Create Representations, Group Presentation

In this activity, you have described patterns and made conjectures about how these patterns would extend beyond your observed cases, based on collected data. Some of your conjectures were probably shown to be untrue when additional data were collected. Other conjectures took on a greater sense of certainty as more confirming data were collected.

In mathematics, there are certain methods and rules of argument that mathematicians use to convince someone that a conjecture is true, even for cases that extend beyond the observed data set. These rules are called rules of logical reasoning or rules of deductive reasoning. An argument that follows such rules is called a proof. A statement or conjecture that has been proven, that is, established as true without a doubt, is called a theorem. A proof transforms a conjecture into a theorem.

Below are some definitions from arithmetic.

Even integer: An integer that has a remainder of 0 when it is divided by 2.
Odd integer: An integer that has a remainder of 1 when it is divided by 2.

Express regularity in repeated reasoning. In the following items, you will make some conjectures about the sums of even and odd integers.

1. Calculate the sum of some pairs of even integers. Show the examples you use and make a conjecture about the sum of two even integers.

2. Calculate the sum of some pairs of odd integers. Show the examples you use and make a conjecture about the sum of two odd integers.

3. Calculate the sum of pairs of integers consisting of one even integer and one odd integer. Show the examples you use and make a conjecture about the sum of an even integer and an odd integer.
Lesson 2-2
Deductive Reasoning

The following items will help you write a convincing argument (a proof) that supports each of the conjectures you made in Items 1–3.

4. Figures A, B, C, and D are puzzle pieces. Each figure represents an integer determined by counting the square pieces in the figure. Use these figures to answer the following questions.

   a. Which of the figures can be used to model an even integer?

   b. Which of the figures can be used to model an odd integer?

   c. Compare and contrast the models of even and odd integers.

5. Model with mathematics. Which pairs of puzzle pieces can fit together to form rectangles? Make sketches to show how they fit.
6. Explain how the figures (when used as puzzle pieces) can be used to show that each of the conjectures in Items 1 through 3 is true.

In Items 5 and 6, you proved the conjectures in Items 1 through 3 geometrically. In Items 7 through 9, you will prove the same conjectures algebraically.

This is an algebraic definition of even integer: An integer is even if and only if it can be written in the form $2p$, where $p$ is an integer. (You can use other variables, such as $2m$, to represent an even integer, where $m$ is an integer.)

7. Reason abstractly. Use the expressions $2p$ and $2m$, where $p$ and $m$ are integers, to confirm the conjecture that the sum of two even integers is an even integer.

This is an algebraic definition of odd integer: An integer is odd if and only if it can be written in the form $2t + 1$, where $t$ is an integer. (Again, you do not have to use $t$ as the variable.)

8. Use expressions for odd integers to confirm the conjecture that the sum of two odd integers is an even integer.
9. Use expressions for even and odd integers to confirm the conjecture that the sum of an even integer and an odd integer is an odd integer.

In Items 5 and 6, you developed a geometric puzzle-piece argument to confirm conjectures about the sums of even and odd integers. In Items 7–9, you developed an algebraic argument to confirm these same conjectures. Even though one method is considered geometric and the other algebraic, they are often seen as the same basic argument.

10. **Construct viable arguments.** Explain the link between the geometric and the algebraic methods of the proof.

11. State three theorems that you have proved about the sums of even and odd integers.
Check Your Understanding

12. Compare and contrast a theorem and a conjecture.

13. Todd knows that 3, 5, 7, 11, and 13 are prime numbers. Based on this evidence, he concludes that all prime numbers are odd.
   a. Is this an example of inductive or deductive reasoning? Explain.
   b. Is Todd’s conclusion correct? Support your answer.

14. Shayla knows that all rectangles have four right angles. She also knows that figure $ABCD$ is a rectangle. She concludes that figure $ABCD$ has four right angles.
   a. Is this an example of inductive or deductive reasoning? Explain.
   b. Critique the reasoning of others. Is Shayla’s conclusion correct? Support your answer.

LESSON 2-2 PRACTICE

15. Use expressions for even and odd integers to confirm the conjecture that the product of an even integer and an odd integer is an even integer.

16. Reason abstractly. Prove this conjecture geometrically: Any odd integer can be expressed as the sum of an even integer and an odd integer.

17. Use deductive reasoning to prove that the solution of the equation $x - 5 = -2$ is $x = 3$. Be sure to justify each step in your proof.

18. Reason quantitatively. Based solely on the pattern in the table, Andre states that the number of sides of a polygon is equal to its number of angles. Is Andre’s statement a conjecture or a theorem? Explain.

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Sides</th>
<th>Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

19. Hairs found at a crime scene are consistent with those of a suspect. Based on this evidence, an investigator concludes that the suspect was at the crime scene. Is this an example of inductive or deductive reasoning? Explain.
ACTIVITY 2 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 2-1

1. Use inductive reasoning to determine the next two terms in the sequence.
   a. 1, 3, 7, 15, 31, …
   b. 3, −6, 12, −24, 48, …

2. Write the first five terms of two different sequences for which 24 is the third term.

3. Generate a sequence using this description: The first term in the sequence is 2, and the terms increase by consecutive odd numbers beginning with 3.

4. Use this picture pattern.

   . . . . . . .
   . . . . . . .
   a. Draw the next shape in the pattern.
   b. Write a sequence of numbers that could be used to express the pattern.
   c. Verbally describe the pattern of the sequence.

5. Two shapes are missing from the sequence below.

   O O ? O O ?

   a. Based on the given shapes, make a conjecture about the missing shapes. Explain the pattern you used to make your conjecture.
   b. Now suppose that you learn that the first missing shape is a triangle. Would this additional information change your conjecture about the other missing shape? Explain.

6. Describe a situation from your everyday life in which you applied inductive reasoning.

7. Kristen divides each convex polygon below into triangles by drawing all diagonals from a single vertex. She conjectures that the number of triangles is always two less than the number of sides of the convex polygon.

   a. Is Kristen’s conjecture reasonable? Explain.
   b. Provide an additional example that supports your answer to Item 7a.

8. Which rule describes how to find the next term of the following sequence?

   64, 40, 28, 22, 19, …

   A. Subtract 24 from the previous term.
   B. Divide the previous term by 2 and add 8.
   C. Divide the previous term by 4 and add 16.
   D. Multiply the previous term by 5 and divide by 8.

9. Explain how you know that the rules you did not choose in Item 8 are incorrect.

10. Use this picture pattern.

   a. Draw the next shape in the pattern.
   b. Write a sequence of numbers that could be used to express the pattern.
   c. Verbally describe the pattern of the sequence.
Lesson 2-2

11. Use expressions for odd integers to confirm the conjecture that the product of two odd integers is an odd integer.

12. For the first four weeks of school, the cafeteria served either spaghetti or lasagna on Thursday. Based on this evidence, Liam states that the cafeteria will serve Italian food next Thursday. Is Liam’s statement a conjecture or a theorem? Explain.

13. Use deductive reasoning to prove that \( x = -3 \) is not in the solution set of the inequality \( -4x < 8 \). Be sure to justify each step in your proof.

14. David notices this pattern.

\[
\begin{align*}
19 &= 1 \times 9 + 1 + 9 \\
29 &= 2 \times 9 + 2 + 9 \\
39 &= 3 \times 9 + 3 + 9
\end{align*}
\]

Based on this pattern, David concludes that any two-digit number ending in 9 is equal to \( n \times 9 + n + 9 \), where \( n \) is the tens digit of the number.

a. Is this an example of inductive or deductive reasoning? Explain.

b. Is David’s conclusion correct? Support your answer.

15. The density of gold is 19.3 grams per cubic centimeter. Rachel determines that the density of a coin is 18.7 grams per cubic centimeter. Based on this evidence, she concludes that the coin is not gold or at least not entirely gold. Is this an example of inductive or deductive reasoning? Explain.

16. Consider the expression \( 3p + 5 \). Bethany states that this expression is even for any integer \( p \) because \( 3p + 5 = 8 \) when \( p = 1 \) and \( 3p + 5 = 20 \) when \( p = 5 \). Is Bethany’s conclusion correct? Support your answer.

17. Consider these true statements.

All amphibians are cold blooded.
All frogs are amphibians.
Chris has a pet frog.

Based on deductive reasoning, which of the following statements is not necessarily true?
A. Chris has a cold-blooded pet.
B. Chris has a pet amphibian.
C. All frogs are cold blooded.
D. All amphibians are frogs.

18. Consider this conjecture: An integer that is divisible by 9 is also divisible by 3.

a. Prove the conjecture geometrically. (Hint: You can represent an integer divisible by 9 by using a rectangle composed of groups of 9 squares, as shown below.)

b. Prove the conjecture algebraically.

c. Explain the link between the geometric and the algebraic methods of the proof.

MATHEMATICAL PRACTICES

Construct Viable Arguments and Critique the Reasoning of Others

19. a. Construct a viable argument for this conjecture: If two angles are complementary, both angles must be acute.

b. What type of reasoning—inductive or deductive—did you use in your argument? Explain.

c. Compare your argument with the argument another student in your group or class has written. Do you agree with this student’s argument? Why or why not?
The Axiomatic System of Geometry
Back to the Beginning
Lesson 3-1 Geometric Definitions and Two-Column Proofs

Learning Targets:
• Distinguish between undefined and defined terms.
• Use properties to complete algebraic two-column proofs.

SUGGESTED LEARNING STRATEGIES: Close Reading, Quickwrite, Think-Pair-Share, Vocabulary Organizer, Interactive Word Wall, Group Presentation, Discussion Groups, Self Revision/Peer Revision

Geometry is an axiomatic system. That means that from a small, basic set of agreed-upon assumptions and premises, an entire structure of logic is devised. Many interactive computer games are designed with this kind of structure. A game may begin with a basic set of scenarios. From these scenarios, a gamer can devise tools and strategies to win the game.

In geometry, it is necessary to agree on clear-cut meanings, or definitions, for words used in a technical manner. For a definition to be helpful, it must be expressed in words whose meanings are already known and understood.

Compare the following definitions.

Fountain: a roundel that is barry wavy of six argent and azure
Guige: a belt that is worn over the right shoulder and used to support a shield

1. Which of the two definitions above is easier to understand? Why?

For a new vocabulary term to be helpful, it should be defined using words that have already been defined. The first definitions used in building a system, however, cannot be defined in terms of other vocabulary words, because no other vocabulary words have been defined yet. In geometry, it is traditional to start with the simplest and most fundamental terms—without trying to define them—and use these terms to define other terms and develop the system of geometry. These fundamental undefined terms are point, line, and plane.

2. Define each term using the undefined terms.
   a. Ray

   b. Collinear points

   c. Coplanar points

The term line segment can be defined in terms of undefined terms: A line segment is part of a line bounded by two points on the line called endpoints.
After a term has been defined, it can be used to define other terms. For example, an *angle* is defined as a figure formed by two rays with a common endpoint.

3. Define each term using the already defined terms.
   a. Complementary angles
   b. Supplementary angles

The process of deductive reasoning, or deduction, must have a starting point. A conclusion based on deduction cannot be made unless there is an established assertion to work from. To provide a starting point for the process of deduction, a number of assertions are accepted as true without proof.

When you solve algebraic equations, you are using deduction. You can use properties to support your reasoning without having to prove that the properties are true.

4. Using one operation or property per step, show how to solve the equation $4x + 9 = 18 - \frac{1}{2}x$. Name each operation or property used to justify each step.
You can organize the steps and the reasons used to justify the steps in two columns with statements (steps) on the left and reasons (properties) on the right. This format is called a **two-column proof**.

### Example A

**Given:** \(3(x + 2) - 1 = 5x + 11\)

**Prove:** \(x = -3\)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (3(x + 2) - 1 = 5x + 11)</td>
<td>1. Given equation</td>
</tr>
<tr>
<td>2. (3(x + 2) = 5x + 12)</td>
<td>2. Addition Property of Equality</td>
</tr>
<tr>
<td>3. (3x + 6 = 5x + 12)</td>
<td>3. Distributive Property</td>
</tr>
<tr>
<td>4. (6 = 2x + 12)</td>
<td>4. Subtraction Property of Equality</td>
</tr>
<tr>
<td>5. (-6 = 2x)</td>
<td>5. Subtraction Property of Equality</td>
</tr>
<tr>
<td>6. (-3 = x)</td>
<td>6. Division Property of Equality</td>
</tr>
<tr>
<td>7. (x = -3)</td>
<td>7. Symmetric Property of Equality</td>
</tr>
</tbody>
</table>

### Try These A

**a.** Supply the reasons to justify each statement in the proof below.

**Given:** \(\frac{x - 3}{2} = \frac{6 + x}{5}\)

**Prove:** \(x = 9\)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (\frac{x - 3}{2} = \frac{6 + x}{5})</td>
<td>1. (\frac{6 + x}{5})</td>
</tr>
<tr>
<td>2. (10\left(\frac{x - 3}{2}\right) = 10\left(\frac{6 + x}{5}\right))</td>
<td>2. (\frac{6 + x}{5})</td>
</tr>
<tr>
<td>3. (5(x - 3) = 2(6 + x))</td>
<td>3. (\frac{6 + x}{5})</td>
</tr>
<tr>
<td>4. (5x - 15 = 12 + 2x)</td>
<td>4. (\frac{6 + x}{5})</td>
</tr>
<tr>
<td>5. (3x - 15 = 12)</td>
<td>5. (\frac{6 + x}{5})</td>
</tr>
<tr>
<td>6. (3x = 27)</td>
<td>6. (\frac{6 + x}{5})</td>
</tr>
<tr>
<td>7. (x = 9)</td>
<td>7. (\frac{6 + x}{5})</td>
</tr>
</tbody>
</table>

**b.** Complete the *Prove* statement and write a two-column proof for the equation given in Item 4. Number each statement and corresponding reason.

**Given:** \(4x + 9 = 18 - \frac{1}{2}x\)

**Prove:**
Lesson 3-1
Geometric Definitions and Two-Column Proofs

Check Your Understanding

5. Explain why undefined terms are necessary in geometry.

6. **Express regularity in repeated reasoning.** What is the relationship between a conjecture, a theorem, and a two-column proof?

7. Jeffrey wrote a two-column proof to solve the equation
   \[ 3(x + 5) = -6x + 6 \]
   In addition to an incorrectly written Prove statement, what error did Jeffrey make in his proof? Rewrite the proof so that it correctly shows how to solve the given equation.

   **Given:** \( 3(x + 5) = -6x + 6 \)  
   **Prove:** \( x = -81 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( 3(x + 5) = -6x + 6 )</td>
<td>1. Given equation</td>
</tr>
<tr>
<td>2. ( 3x + 15 = -6x + 6 )</td>
<td>2. Distributive Property</td>
</tr>
<tr>
<td>3. ( 9x + 15 = 6 )</td>
<td>3. Addition Property of Equality</td>
</tr>
<tr>
<td>4. ( 9x = -9 )</td>
<td>4. Subtraction Property of Equality</td>
</tr>
<tr>
<td>5. ( x = -81 )</td>
<td>5. Multiplication Property of Equality</td>
</tr>
</tbody>
</table>

**LESSON 3-1 PRACTICE**

8. Identify the property that justifies the statement: If \( 4x - 3 = 7 \), then \( 4x = 10 \).

9. Complete the prove statement and write a two-column proof for the equation:
   \[ x - 2 = 3(x - 4) \]
   **Prove:**

10. **Construct viable arguments.** Explain why *line segment* is considered a defined term in geometry.

11. Complete the prove statement and write a two-column proof for the equation:
   \[ 2n - 21 = \frac{n}{4} \]
   **Prove:**

12. **Look for and make use of structure.** Suppose you are given that \( a = b + 2 \) and \( b + 2 = 5 \). What can you prove by using these statements and the Transitive Property?
Lesson 3-2
Conditional Statements

Learning Targets:
• Identify the hypothesis and conclusion of a conditional statement.
• Give counterexamples for false conditional statements.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall, Marking the Text, Self Revision/Peer Revision, Discussion Groups

Rules of logical reasoning involve using a set of given statements along with a valid argument to reach a conclusion. Statements to be proved are often written in if-then form. An if-then statement is called a conditional statement. In such statements, the if clause is the hypothesis, and the then clause is the conclusion.

Example A

Conditional statement: If $3(x + 2) - 1 = 5x + 11$, then $x = -3$.

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3(x + 2) - 1 = 5x + 11$</td>
<td>$x = -3$</td>
</tr>
</tbody>
</table>

Try These A

Use the conditional statement: If $x + 7 = 10$, then $x = 3$.

a. What is the hypothesis?

b. What is the conclusion?

c. State the property of equality that justifies the conclusion of the statement.

Conditional statements may not always be written in if-then form. You can restate such conditional statements in if-then form.

1. **Make use of structure.** Restate each conditional statement in if-then form.
   a. I'll go if you go.

   b. There is smoke only if there is fire.

   c. $x = 4$ implies $x^2 = 16$.

The letters $p$ and $q$ are often used to represent the hypothesis and conclusion, respectively, in a conditional statement. The basic form of an if-then statement would then be, “If $p$, then $q$.”

**Forms of conditional statements** include:
- If $p$, then $q$.
- $p$ only if $q$
- $p$ implies $q$.
- $q$ if $p$.
An if-then statement is false if an example can be found for which the hypothesis is true and the conclusion is false. This type of example is a **counterexample**.

2. This is a false conditional statement.
   
   *If two numbers are odd, then their sum is odd.*
   
   a. Identify the hypothesis of the statement.
   
   b. Identify the conclusion of the statement.
   
   c. Give a counterexample for the conditional statement and justify your choice for this example.

**Check Your Understanding**

3. **Reason abstractly.** If you can find an example for which both the hypothesis and the conclusion of a conditional statement are true, is the conditional statement itself necessarily true? Explain.

4. Give an example of a true conditional statement that includes this hypothesis: An angle is named \( \angle ABC \).

5. Give an example of a true conditional statement that includes this conclusion: The angles share a vertex.

6. Cesar conjectures that the quotient of any two even numbers greater than 0 is odd.
   a. Write Cesar’s conjecture as a conditional statement.
   b. Give a counterexample to show that Cesar’s conjecture is false.

7. Write the definition of **collinear points** as a conditional statement.
Lesson 3-2
Conditional Statements

LESSON 3-2 PRACTICE

8. Write the statement in if-then form:
   Two angles have measures that add up to 90° only if they are complements of each other.

9. Which of the following is a counterexample of this statement?
   If an angle is acute, then it measures 80°.
   A. a 100° angle    B. a 90° angle
   C. an 80° angle    D. a 70° angle

10. Identify the hypothesis and the conclusion of the statement:
    If it is not raining, then I will go to the park.

11. Critique the reasoning of others. Joanna says that 4 + 7 = 11 is a counterexample that shows that the following conditional statement is false. Is Joanna correct? Explain.
    If two integers are even, then their sum is even.

12. Construct viable arguments. Why do you only need a single counterexample to show that a conditional statement is false?
Learning Targets:
- Write and determine the truth value of the converse, inverse, and contrapositive of a conditional statement.
- Write and interpret biconditional statements.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall, Think-Pair-Share, Group Presentation, Discussion Groups

Every conditional statement has three related conditionals. These are the converse, the inverse, and the contrapositive of the conditional statement. The converse of a conditional is formed by interchanging the hypothesis and conclusion of the statement. The inverse is formed by negating both the hypothesis and the conclusion of the statement. Finally, the contrapositive is formed by interchanging and negating both the hypothesis and the conclusion.

Conditional: If $p$, then $q$.
Converse: If $q$, then $p$.
Inverse: If not $p$, then not $q$.
Contrapositive: If not $q$, then not $p$.

1. Given the conditional statement:
   
   If a figure is a triangle, then it is a polygon.

Complete the table.

<table>
<thead>
<tr>
<th>Form of the statement</th>
<th>Write the statement</th>
<th>True or False?</th>
<th>If the statement is false, give a counterexample.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional statement</td>
<td>If a figure is a triangle, then it is a polygon.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Converse of the conditional statement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inverse of the conditional statement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contrapositive of the conditional statement</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Lesson 3-3
Converse, Inverse, and Contrapositive

If a given conditional statement is true, the converse and inverse are not necessarily true. However, the contrapositive of a true conditional is always true, and the contrapositive of a false conditional is always false. Likewise, the converse and inverse of a conditional are either both true or both false. Statements with the same truth values are logically equivalent.

2. Write a true conditional statement whose inverse is false.

3. Write a true conditional statement that is logically equivalent to its converse.

When a statement and its converse are both true, they can be combined into one statement using the words “if and only if.” An “if and only if” statement is a biconditional statement. All definitions you have learned can be written as biconditional statements.

4. Write the definition of perpendicular lines in biconditional form.

5. Consider the statement: Numbers that do not end in 2 are not even.
   a. Rewrite the statement in if-then form and state whether it is true or false.

   b. Write the converse and state whether it is true or false. If false, give a counterexample.

   c. Write the inverse and state whether it is true or false.

   d. Write the contrapositive and state whether it is true or false. If false, give a counterexample.

   e. Can you write a biconditional statement for the original statement? Why or why not?
Check Your Understanding

6. Write four true conditional statements based on this biconditional statement.
   \[
   \text{An angle is a right angle if and only if it measures } 90^\circ.
   \]

7. A conditional statement is true, and its inverse is false. What can you conclude about the converse and the contrapositive of the conditional statement?

8. Make use of structure. Are these two statements logically equivalent? Explain.
   \[
   \text{If a polygon is a square, then it is a quadrilateral.}
   \]
   \[
   \text{If a polygon is a quadrilateral, then it is a square.}
   \]

9. Critique the reasoning of others. Toby says that the converse of the following statement is true. Is Toby’s reasoning correct? Explain.
   \[
   \text{If a number is divisible by 6, then it is divisible by 2.}
   \]

10. Consider this statement.
    \[
    \text{All birds have wings.}
    \]
    a. Write the statement as a conditional statement.
    b. Can you write the statement as a biconditional statement? Explain.

LESSON 3-3 PRACTICE

Use the following statement for Items 11–13.
If a vehicle has four wheels, then it is a car.

11. Write the converse.
12. Write the inverse.
13. Write the contrapositive.
14. Write the definition of the vertex of an angle as a biconditional statement.
15. Give an example of a true statement that has a false converse.
    Given: (1) If X is blue, then Y is gold.
    \[
    \text{(2) Y is not gold.}
    \]
    Which of the following must be true?
    A. Y is blue.
    B. Y is not blue.
    C. X is not blue.
    D. X is gold.
ACTIVITY 3 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 3-1
1. Identify the property that justifies the statement: 
   \[ 5(x - 3) = 5x - 15 \]
   A. multiplication    B. transitive
   C. subtraction    D. distributive
2. a. Give an example of an undefined term and a defined term in geometry.
   b. Explain the difference between an undefined term and a defined term.
3. How is a definition different from a theorem?
4. What are the missing reasons and statements in the two-column proof?

<table>
<thead>
<tr>
<th>Given: ( \frac{2x}{4} = \frac{5x + 8}{2} )</th>
<th>Prove: ( x = -2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Statements</strong></td>
<td><strong>Reasons</strong></td>
</tr>
<tr>
<td>1. ( \frac{2x}{4} = \frac{5x + 8}{2} )</td>
<td>1. Given equation</td>
</tr>
<tr>
<td>2. ( 2x = 2(5x + 8) )</td>
<td>2. ( b. )</td>
</tr>
<tr>
<td>3. ( -2 = x )</td>
<td>3. Distributive Property</td>
</tr>
<tr>
<td>4. ( 0 = 8x + 16 )</td>
<td>4. ( d. )</td>
</tr>
<tr>
<td>5. ( -2 = x )</td>
<td>5. Subtraction Property of Equality</td>
</tr>
<tr>
<td>6. ( -2 = x )</td>
<td>6. ( f. )</td>
</tr>
<tr>
<td>7. ( -2 = x )</td>
<td>7. Symmetric Property</td>
</tr>
</tbody>
</table>

Lesson 3-2
For Items 8–10, write each statement in if-then form.
8. Dianna will go to the movie if she finishes her homework.
9. \( m\angle G = 40^\circ \) implies \( \angle G \) is acute.
10. A figure is a triangle only if it is a polygon with three sides.
11. State the hypothesis and the conclusion of this conditional statement.
   If the temperature drops below 65°F, then the swimming pool closes.
12. Given the false conditional statement, “If a vehicle is built to fly, then it is an airplane,” write a counterexample.
13. Dustin says that \( 8 \cdot -1 = 8 \) is a counterexample that shows that the following conditional statement is false. Is Dustin correct? Explain.
   If two integers are multiplied, then the product is greater than both integers.
14. Given that the hypothesis of the following conditional statement is true, which statement must also be true?
   If Margo wears gloves or a scarf, then she wears a coat.
   A. Margo is wearing a coat.
   B. Margo is wearing gloves.
   C. Margo is not wearing a coat.
   D. Margo is not wearing gloves.
15. a. Write a true conditional statement that includes this hypothesis: \( -3x + 10 = -5 \).
   b. Write a two-column proof to prove that your conditional statement is true.
Lesson 3-3

Use this statement for Items 16–19.

*If today is Thursday, then tomorrow is Friday.*

16. Write the converse of the statement.
17. Write the inverse of the statement.
18. Write the contrapositive of the statement.
19. Can the conditional statement be written as a biconditional statement? If so, write the biconditional statement. If not, explain why not.

20. A certain conditional statement is true. Which of the following must also be true?
   A. converse
   B. inverse
   C. contrapositive
   D. all of the above

21. Give an example of a statement that is false and logically equivalent to its inverse.

22. Compare and contrast a true conditional statement and a biconditional statement.

For Items 23–26, tell whether each statement is true or false. If it is false, give a counterexample.

23. If a number is a multiple of 8, then it is a multiple of 4.
24. the converse of the statement in Item 23
25. the inverse of the statement in Item 23
26. the contrapositive of the statement in Item 23

27. The following statement is the contrapositive of a conditional statement. What is the original conditional statement?

   *If a parallelogram does not have four right angles, then it is not a rectangle.*

28. Consider this conditional statement.

   *If two numbers are negative, then their sum is less than both numbers.*

   Lorenzo says that the inverse of the statement can be written as follows.

   *If two numbers are positive, then their sum is greater than both numbers.*

   Is Lorenzo’s reasoning correct? Explain.

Use this statement for Items 29–32.

*If the sum of the measures of two angles is 90°, then the angles are complementary.*

29. Write the inverse of the converse of the statement.
30. What is another name for the inverse of the converse?
31. Write the contrapositive of the inverse of the statement.
32. What is another name for the contrapositive of the inverse?

MATHEMATICAL PRACTICES

Attend to Precision

33. Write a clear definition of the term adjacent angles. Then use your definition to explain why ∠KJL and ∠LJM are adjacent angles but ∠KJL and ∠KJM are not adjacent angles.
Origami is the art of paper folding. In traditional origami, a single sheet of paper is folded to create a three-dimensional design, such as a crane, a dragon, or a flower. Origami also has practical applications. For example, origami has been used to fold car airbags efficiently so that they will inflate correctly in an accident.

Origami designs make use of geometric figures such as points, lines, rays, segments, and angles.

1. The diagrams at the right show how you can use origami to construct an equilateral triangle from a rectangular sheet of paper. Describe each step in words as precisely as possible, using correct geometric notation. The first step has been done for you.

   **Step 1:** Fold rectangle $ABCD$ along a line so that $AB$ aligns on top of $CD$. Then unfold. The fold creates $EF$.

2. The diagram below shows $\triangle GAD$ from the beginning of Step 3. Based on this diagram, $m\angle AGD + 90^\circ + 30^\circ = 180^\circ$.

   Complete the Prove statement and write a two-column proof for the equation.

   **Given:** $m\angle AGD + 90^\circ + 30^\circ = 180^\circ$

   **Prove:**

3. Consider this statement: “A triangle is equilateral if it has three sides with the same length.”

   a. Write the statement in if-then form. Identify the hypothesis and conclusion.

   b. Write the converse, inverse, and contrapositive of the conditional statement. Which are true?

   c. Can the conditional statement be written as a biconditional statement? Explain.
4. Suppose you start with a stack of printer paper that is 0.02 inch thick. The table shows the thickness of the stack after folding the paper in half several times.

<table>
<thead>
<tr>
<th>Number of Folds</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness (in.)</td>
<td>0.04</td>
<td>0.08</td>
<td>0.16</td>
<td>0.32</td>
<td>0.64</td>
</tr>
</tbody>
</table>

a. Make a conjecture about the thickness of the stack after folding the paper in half six times. Explain the pattern you used to determine your answer.

b. Did you use inductive or deductive reasoning to make your conjecture? Explain.

c. Ellis claims that he could fold a sheet of printer paper in half 25 times. Is his claim realistic? Support your answer.
Learning Targets:
- Apply the Segment Addition Postulate to find lengths of segments.
- Use the definition of midpoint to find lengths of segments.

SUGGESTED LEARNING STRATEGIES: Close Reading, Look for a Pattern, Think-Pair-Share, Vocabulary Organizer, Interactive Word Wall, Create Representations, Marking the Text, Visualization, Identify a Subtask, Discussion Groups

In geometry, axioms, or postulates, are statements that are accepted as true without proof in order to provide a starting point for deductive reasoning. Like point, line, and plane, distance along a line is an undefined term in geometry used to define other geometric terms. For example, the length of a line segment is the distance between its endpoints.

If two points are no more than 1 foot apart, you can find the distance between them by using an ordinary ruler. (The inch rulers below have been reduced to fit on the page.)

In the figure, the distance between point A and point B is 5 inches. Of course, there is no need to place the zero of the ruler on point A. In the figure below, the 2-inch mark is on point A. In this case, AB, measured in inches, is |7 − 2| = |2 − 7| = 5, as before.

The number obtained as a measure of distance depends on the unit of length. For example, the distance between two points in inches will be a different number than the distance between the two points in centimeters.

1. Determine the length of each segment in centimeters.

   a. \( DE = \)   
   b. \( EF = \)   
   c. \( DF = \)
2. **Attend to precision.** Determine the length of each segment in centimeters (to the nearest tenth).

\[ \begin{align*}
G \quad & \quad H \quad & \quad K \\
\end{align*} \]

a. \( KH = \)  

b. \( HG = \)  

c. \( GK = \)  

3. Using your results from Items 1 and 2, describe any patterns that you notice.

4. Given that \( N \) is a point between endpoints \( M \) and \( P \) of line segment \( MP \), describe how to determine the length of \( MP \), without measuring, if you are given the lengths of \( MN \) and \( NP \).

5. Use the Segment Addition Postulate and the given information to complete each statement.

a. If \( B \) is between \( C \) and \( D \), \( BC = 10 \text{ in.} \), and \( BD = 3 \text{ in.} \), then \( CD = \)______.

b. If \( Q \) is between \( R \) and \( T \), \( RT = 24 \text{ cm} \), and \( QR = 6 \text{ cm} \), then \( QT = \)______.

c. If \( P \) is between \( L \) and \( A \), \( PL = x + 4 \), \( PA = 2x - 1 \), and \( LA = 5x - 3 \), then \( x = \)______ and \( LA = \)______.
Lesson 4-1
Segments and Midpoints

The **midpoint** of a segment is the point on the segment that divides it into two **congruent** segments. For example, if \( B \) is the midpoint of \( \overline{AC} \), then \( AB \cong BC \).

![Diagram of a segment with points A, B, and C]

6. Given: \( M \) is the midpoint of \( \overline{RS} \). Complete each statement.
   
   a. If \( RS = 10 \), then \( SM = \) ________.
   
   b. If \( RM = 12 \), then \( MS = \) ________, and \( RS = \) ________.

---

Check Your Understanding

7. Points \( D \) and \( E \) are aligned with a ruler. Point \( D \) is at the mark for 4.5 cm, and the distance between points \( D \) and \( E \) is 3.4 cm. At which two marks on the ruler could point \( E \) be located?

8. Point \( N \) is the midpoint of \( \overline{FG} \). If \( FN = 2x \), what expression represents \( FG \)?

9. **Reason abstractly.** Does a ray have a midpoint? Explain.

10. Give an example that illustrates the Segment Addition Postulate. Include a sketch with your example.

---

You can also use a number line to find the distance between two points.

11. What is \( LM \)?

![Number line with points L and M]
The midpoint of a segment is halfway between its endpoints. So, if you know the coordinates of the endpoints, you can average them to find the coordinate of the midpoint.

12. What is the coordinate of the midpoint \( M \) of \( PQ \)?

![Coordinate Plane]

13. Use the number line to solve each problem.

![Coordinate Plane]

**Check Your Understanding**

13. Use the number line to solve each problem.

**Math Tip**

A number line represents a one-dimensional coordinate system. You will explore the concepts of distance and midpoint using a two-dimensional coordinate system when you work with the coordinate plane in the next activity.
Lesson 4-1
Segments and Midpoints

You can use the definition of midpoint and properties of algebra to determine the length of a segment.

Example A
If Q is the midpoint of \( PR \), \( PQ = 4x - 5 \), and \( QR = 11 + 2x \), determine the length of \( PQ \).

Because \( Q \) is the midpoint of \( PR \), you know that \( PQ \cong QR \) and \( PQ = QR \).

\[
PQ = QR
\]
\[
4x - 5 = 11 + 2x \quad \text{Substitution Property}
\]
\[
2x - 5 = 11 \quad \text{Subtraction Property of Equality}
\]
\[
2x = 16 \quad \text{Addition Property of Equality}
\]
\[
x = 8 \quad \text{Division Property of Equality}
\]

Now substitute 8 for \( x \) in the expression for \( PQ \).

\[
PQ = 4x - 5 = 4(8) - 5 = 27
\]

Try These A

a. If \( Y \) is the midpoint of \( WZ \), \( YZ = x + 3 \), and \( WZ = 3x - 4 \), determine the length of \( WZ \).

Given: \( M \) is the midpoint of \( RS \). Use the given information to find the missing values.

b. \( RM = x + 3 \) and \( MS = 2x - 1 \)

\[
x = \ldots \quad \text{and} \quad RM = \ldots
\]

c. \( RM = x + 6 \) and \( RS = 5x + 3 \)

\[
x = \ldots \quad \text{and} \quad SM = \ldots
\]

When you bisect a geometric figure, you divide it into two equal or congruent parts.

14. Reason abstractly. Line segment \( WZ \) bisects \( XY \) at point \( Z \). What are two conclusions you can draw from this information?
15. Explain how to find the distance between two points on a number line.

16. Mekhi knows that \( GH = 7 \) and \( HJ = 7 \). Based on this information, he claims that point \( H \) is the midpoint of \( GJ \). Is Mekhi’s claim necessarily true? Make a sketch that supports your answer.

17. Given: \( T \) is the midpoint of \( JK \), \( JK = 5x - 3 \), and \( JT = 2x + 1 \). Determine the length of \( JK \).

18. Reason quantitatively. \( AB \) lies on a number line. The coordinate of point \( A \) is \(-6\). Given that \( AB = 20 \), what are the two possible coordinates for point \( B \)?

**LESSON 4-1 PRACTICE**

19. Given: Point \( K \) is between points \( H \) and \( J \), \( HK = x - 5 \), \( KJ = 5x - 12 \), and \( HJ = 25 \). Find the value of \( x \).

20. If \( B \) is the midpoint of \( AC \), \( AB = x + 6 \), and \( AC = 5x - 6 \), then what is \( BC \)?

21. Point \( P \) is between points \( F \) and \( G \). The distance between points \( F \) and \( P \) is \( \frac{1}{4} \) of \( FG \). What is the coordinate of point \( P \)?

22. Use appropriate tools strategically. Anne has a broken ruler. It starts at the 3-inch mark and ends at the 12-inch mark. Explain how Anne could use the ruler to measure the length of a line segment in inches.

23. If \( P \) is the midpoint of \( ST \), \( SP = x + 4 \), and \( ST = 4x \), determine the length of \( ST \).

24. Compare and contrast a postulate and a conjecture.
Learning Targets:
- Apply the Angle Addition Postulate to find angle measures.
- Use the definition of angle bisector to find angle measures.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Look for a Pattern, Quickwrite, Vocabulary Organizer, Interactive Word Wall, Create Representations, Debriefing, Identify a Subtask, Group Presentation

You measure angles with a protractor. The number of degrees in an angle is called its measure.

You measure angles with a protractor. The number of degrees in an angle is called its measure.

1. **Use appropriate tools strategically.** Determine the measure of each angle.
   - a. \( m\angle AOB = 50^\circ \)
   - b. \( m\angle BOC = \) ___
   - c. \( m\angle AOC = \) ___
   - d. \( m\angle EOD = \) ___
   - e. \( m\angle BOD = \) ___
   - f. \( m\angle BOE = \) ___

2. Use a protractor to determine the measure of each angle.
   - a. \( m\angle TQP = \) ___
   - b. \( m\angle TQR = \) ___
   - c. \( m\angle RQP = \) ___

3. **Express regularity in repeated reasoning.** Using your results from Items 1 and 2, describe any patterns that you notice.

4. Given that point \( D \) is in the interior of \( \angle ABC \), describe how to determine the measure of \( \angle ABC \), without measuring, if you are given the measures of \( \angle ABD \) and \( \angle DBC \).

CONNECT TO ASTRONOMY

The astronomer Claudius Ptolemy (about 85–165 CE) based his observations of the solar system on a unit that resulted from dividing the distance around a circle into 360 parts. This later became known as a degree.

MATH TERMS

**The Protractor Postulate**

a. To each angle there corresponds a unique real number between 0 and 180 called the measure of the angle.

b. The measure of an angle formed by a pair of rays is the absolute value of the difference of their associated numbers.

**MATH TERMS**

Item 4 and your answer together form a statement of the Angle Addition Postulate.
5. Use the Angle Addition Postulate and the given information to complete each statement.

a. If \( P \) is in the interior of \( \angle XYZ \), \( m\angle XYP = 25^\circ \), and \( m\angle PYZ = 50^\circ \), then \( m\angle XYZ = \) ______.

b. If \( M \) is in the interior of \( \angle RTD \), \( m\angle RTM = 40^\circ \), and \( m\angle RTD = 65^\circ \), then \( m\angle MTD = \) ______.

c. If \( H \) is in the interior of \( \angle EFG \), \( m\angle EFH = 75^\circ \), and \( m\angle HFG = (10x)^\circ \), and \( m\angle EFG = (20x - 5)^\circ \), then \( x = \) ______ and \( m\angle HFG = \) ______.

d. Lines \( DB \) and \( EC \) intersect at point \( F \). If \( m\angle BFC = 44^\circ \) and \( m\angle AFB = 61^\circ \), then
   \( m\angle AFC = \) ______
   \( m\angle AFE = \) ______
   \( m\angle EFD = \) ______

6. Given: \( AH \) bisects \( \angle MAT \). Determine the missing measure.

a. \( m\angle MAT = 70^\circ \), \( m\angle MAH = \) ______

b. \( m\angle HAT = 80^\circ \), \( m\angle MAT = \) ______.
Lesson 4-2
Angles and Angle Bisectors

You can use definitions, postulates, and properties of algebra to determine the measures of angles.

Example A
If \( \overline{QP} \) bisects \( \angle DQL \), \( m \angle DQP = 5x - 7 \), and \( m \angle PQL = 11 + 2x \), determine the measure of \( \angle DQL \).

Because \( \overline{QP} \) is the angle bisector of \( \angle DQL \), you know that \( \angle DQP \cong \angle PQL \) and \( m \angle DQP = m \angle PQL \).

\[
\begin{align*}
5x - 7 &= 11 + 2x \\
3x - 7 &= 11 \\
3x &= 18 \\
x &= 6
\end{align*}
\]

Substitution Property
Subtraction Property of Equality
Addition Property of Equality
Division Property of Equality

Substitute 6 for \( x \) in the expressions for \( m \angle DQP \) and \( m \angle PQL \).

\[
\begin{align*}
m \angle DQP &= 5(6) - 7 = 23^\circ \\
m \angle PQL &= 11 + 2(6) = 23^\circ
\end{align*}
\]

By the Angle Addition Postulate, \( m \angle DQL = m \angle DQP + m \angle PQL \), so \( m \angle DQL = 23^\circ + 23^\circ = 46^\circ \).

Make a sketch of \( \angle DQL \) and its angle bisector \( \overline{QP} \). Label the measures of \( \angle DQP \) and \( \angle PQL \).

Try These A
Given: \( \overline{FL} \) bisects \( \angle AFM \). Determine each missing value.

a. \( m \angle LFM = 11x + 4 \) and \( m \angle AFL = 12x - 2 \)
   
   \( x = \)_______, \( m \angle LFM = \)_______, and \( m \angle AFM = \)_______

b. \( m \angle AFM = 6x - 2 \) and \( m \angle AFL = 4x - 10 \)
   
   \( x = \)_______ and \( m \angle LFM = \)_______

7. \( \angle A \) and \( \angle B \) are complementary, \( m \angle A = 3x + 7 \), and \( m \angle B = 6x + 11 \). Determine the measure of each angle.
8. In the diagram at the left, $AC$ and $DB$ intersect as shown. Determine the measure of $\angle CEB$.

Check Your Understanding

9. $KN$ is the angle bisector of $\angle JKL$. Explain how you could find $m\angle KN$ if you know $m\angle JKL$.

10. In this diagram, $m\angle 1 = 4x + 30$ and $m\angle 3 = 2x + 48$. Write a step-by-step explanation for an absent classmate showing how to find $m\angle 2$.

11. **Construct viable arguments.** $\angle RST$ is adjacent to $\angle TSU$, $m\angle RST = 40^\circ$, and $m\angle TSU = 50^\circ$. Explain how you can use the Angle Addition Postulate to show that $\angle RSU$ is a right angle.

**LESSON 4-2 PRACTICE**

12. Point $D$ is in the interior of $\angle ABC$, $m\angle ABC = 10x - 7$, $m\angle ABD = 6x + 5$, and $m\angle DBC = 36^\circ$. What is $m\angle ABD$?

13. $QS$ bisects $\angle PQR$. If $m\angle PQS = 5x$ and $m\angle RQS = 2x + 6$, then what is $m\angle PQR$?

14. $\angle L$ and $\angle M$ are complementary, $m\angle L = 2x + 25$, and $m\angle M = 4x + 11$. Determine the measure of each angle.

15. In this diagram, $\overline{EA} \perp \overline{ED}$ and $\overline{EB}$ bisects $\angle AEC$. Given that $m\angle AEB = 4x + 1$ and $m\angle CED = 3x$, determine the missing measures.
   a. $x = \ldots$
   b. $m\angle BEC = \ldots$

16. **Critique the reasoning of others.** Penny knows that point $W$ is in the interior of $\angle XYZ$. Based on this information, she claims that $\angle XYW \cong \angle WYZ$. Is Penny’s claim necessarily true? Explain. Make a sketch that supports your answer.
ACTIVITY 4 PRACTICE
Write your answers on notebook paper.
Show your work.

Lesson 4-1
1. If \( Q \) is between \( A \) and \( M \) and \( MQ = 7.3 \) and \( AM = 8.5 \), then \( QA = ? \).
   A. 5.8  
   B. 1.2  
   C. 7.3  
   D. 14.6

2. Given: \( K \) is between \( H \) and \( J \), \( HK = 2x - 5 \), \( KJ = 3x + 4 \), and \( HJ = 24 \). What is the value of \( x \)?
   A. 9  
   B. 5  
   C. 19  
   D. 3

3. If \( K \) is the midpoint of \( HJ \), \( HK = x + 6 \), and \( HJ = 4x - 6 \), then \( KJ = ? \).
   A. 15  
   B. 9  
   C. 4  
   D. 10

4. State the Segment Addition Postulate in your own words.

5. Explain what distance along a line means as an undefined term in geometry.

Use the number line for Items 6–9.

6. What is \( AB \)?

7. What is the coordinate of the midpoint of \( AB \)? Explain how you found your answer.

8. Point \( M \) is the midpoint of \( AB \). What is the coordinate of the midpoint of \( AM \)?

9. Point \( C \) is between points \( A \) and \( B \). The distance between points \( B \) and \( C \) is \( \frac{1}{4} \) of \( AB \). What is the coordinate of point \( C \)?

10. \( FG \) lies on a number line. The coordinate of point \( F \) is 8. Given that \( FG = 16 \), what are the two possible coordinates for point \( G \)?

11. In the diagram, \( ST \) is aligned with a centimeter ruler. What is the length of \( ST \) in centimeters?

12. Compare and contrast a postulate and a theorem.

13. What are two conclusions you can draw from this statement? Support your answers.
   Point \( P \) is the midpoint of \( QR \).

14. If \( D \) is the midpoint of \( CE \), \( CD = x + 7 \), and \( CE = 5x - 1 \), determine each missing value.
   a. \( x = ? \)  
   b. \( CD = ? \)  
   c. \( CE = ? \)  
   d. \( DE = ? \)

15. Leo is running in a 5-kilometer race along a straight path. If he is at the midpoint of the path, how many kilometers does he have left to run?

16. Point \( S \) is between points \( R \) and \( T \). Given that \( RS \cong ST \) and \( RS = 16 \), what is \( RT \)?

Lesson 4-2

17. \( P \) lies in the interior of \( \angle RST \), \( m\angle RSP = 40^\circ \) and \( m\angle TSP = 10^\circ \). \( m\angle RST = ? \).
   A. 100°  
   B. 50°  
   C. 30°  
   D. 10°

18. \( QS \) bisects \( \angle PQR \). If \( m\angle PQS = 3x \) and \( m\angle RQS = 2x + 6 \), then \( m\angle PQR = ? \).
   A. 18°  
   B. 36°  
   C. 30°  
   D. 6°

19. \( \angle P \) and \( \angle Q \) are supplementary. \( m\angle P = 5x + 3 \) and \( m\angle Q = x + 3 \). \( x = ? \).
   A. 14  
   B. 0  
   C. 29  
   D. 30
20. In this figure, \( m\angle 3 = x + 18 \), \( m\angle 4 = x + 15 \), and \( m\angle 5 = 4x + 3 \). Show all work for parts a, b, and c.

   a. What is the value of \( x \)?
   b. What is the measure of \( \angle 1 \)?
   c. Is \( \angle 3 \) complementary to \( \angle 1 \)? Explain.

21. \( MP \) is the angle bisector of \( \angle L MN \). Given that \( \angle LMP \) is a right angle, what type of angle is \( \angle LM N \)? Explain how you know.

22. State the Angle Addition Postulate in your own words.

23. A protractor is properly aligned with the vertex of \( \angle K LM \). \( LK \) passes through the mark for \( 38^\circ \) on the protractor and \( LM \) passes through the mark for \( 126^\circ \). What is \( m\angle KLM \)?

24. \( \angle GF H \) and \( \angle HF J \) are adjacent and congruent. What are two conclusions you can draw from this information? Support your answers.

25. In this figure, \( m\angle 1 = 4x + 50 \) and \( m\angle 3 = 2x + 66 \). Show all work for parts a, b, and c.

   a. What is the value of \( x \)?
   b. What is the measure of \( m\angle 3 \)?
   c. What is the measure of \( m\angle 2 \)?

26. Given: \( AE \perp AB \), \( AD \) bisects \( \angle EAC \), \( m\angle CAB = 2x - 4 \) and \( m\angle CAE = 3x + 14 \). Show all work for parts a and b.

    a. What is the value of \( x \)?
    b. What is the measure of \( m\angle DAC \)?

MATHEMATICAL PRACTICES
Look for and Express Regularity in Repeated Reasoning

27. \( \angle CDF \) is one of the congruent angles formed by the angle bisector of \( \angle CDE \). Write a formula that can be used to determine \( m\angle CDF \) given \( m\angle CDE \).
Distance and Midpoint Formulas
We ♥ Descartes
Lesson 5-1 Distance on the Coordinate Plane

Learning Targets:
• Derive the Distance Formula.
• Use the Distance Formula to find the distance between two points on the coordinate plane.

SUGGESTED LEARNING STRATEGIES: Create Representations, Simplify the Problem, Think-Pair-Share, Think Aloud, Visualization, Look for a Pattern, Graphic Organizer, Discussion Groups, Identify a Subtask

In the previous activity, you used number lines to determine distance and to locate the midpoint of a segment. A number line is a one-dimensional coordinate system.

In this activity, you will explore the concepts of distance and midpoint on a two-dimensional coordinate system, or coordinate plane.

Use the coordinate plane and follow the steps below to determine the distance between points $P(1, 4)$ and $Q(13, 9)$.

1. **Model with mathematics.** Plot the points $P(1, 4)$ and $Q(13, 9)$ on the coordinate plane. Then draw $PQ$.
2. **Draw horizontal segment $PR$ and vertical segment $QR$ to create right triangle $PQR$, with a right angle at vertex $R$.**
3. **Attend to precision.** What are the coordinates of point $R$?

CONNECT TO HISTORY
The Cartesian coordinate system was developed by the French philosopher and mathematician René Descartes in 1637 as a way to specify the position of a point or object on a plane.

READING MATH
The notation $P(1, 4)$ means point $P$ with coordinates $(1, 4)$ on the coordinate plane.
Lesson 5-1
Distance on the Coordinate Plane

4. a. What is PR, the length of the horizontal leg of the right triangle?
   
   b. What is QR, the length of the vertical leg of the right triangle?
   
   c. Explain how you determined your answers to parts a and b.

5. Use the Pythagorean Theorem to find PQ. Show your work.

6. Attend to precision. What is the distance between points P(1, 4) and Q(13, 9)? How do you know?

7. How do you know that the triangle you drew in Item 2 is a right triangle?

8. What relationship do you notice among the coordinates of points P, Q, and R?

Check Your Understanding

9. Explain how you would find the distance between points J(−3, 6) and K(3, 14).

10. What is the distance between M(9, −5) and N(−11, 10)?

MATH TIP
The Pythagorean Theorem states that the square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs of the right triangle. In other words, for a right triangle with hypotenuse c and legs a and b, \( c^2 = a^2 + b^2 \).
Lesson 5-1
Distance on the Coordinate Plane

Although the method you just learned for finding the distance between two points will always work, it may not be practical to plot points on a coordinate plane and draw a right triangle each time you want to find the distance between them.

Instead, you can use algebraic methods to derive a formula for finding the distance between any two points on the coordinate plane. Start with any two points \((x_1, y_1)\) and \((x_2, y_2)\) on the coordinate plane. Visualize using these points to draw a right triangle with a horizontal leg and a vertical leg.

11. What are the coordinates of the point at the vertex of the right angle of the triangle?

12. Write an expression for the length of the horizontal leg of the right triangle.

13. Write an expression for the length of the vertical leg of the right triangle.

14. Use the Pythagorean Theorem to write an expression for the length of the hypotenuse of the right triangle.

15. Express regularity in repeated reasoning. Write a formula that can be used to find \(d\), the distance between two points \((x_1, y_1)\) and \((x_2, y_2)\) on the coordinate plane.
16. Use the formula you wrote in Item 15 to find the distance between the points with coordinates $(12, -5)$ and $(-3, 7)$.

The coordinate geometry formulas introduced in this activity are used frequently in AP calculus in a wide variety of applications.

**Check Your Understanding**

17. Find the distance between the points with the coordinates shown.
   a. $(-8, 5)$ and $(7, -3)$
   b. $(3, 8)$ and $(8, 3)$

18. Create a Venn diagram that compares and contrasts the Pythagorean Theorem and the Distance Formula.

19. **Reason abstractly.** Suppose two points lie on the same vertical line. Can you use the Distance Formula to find the distance between them? Explain.

20. Write and simplify a formula for the distance $d$ between the origin and a point $(x, y)$ on the coordinate plane.
Lesson 5-1
Distance on the Coordinate Plane

LESSON 5-1 PRACTICE

Find the distance between the points with the given coordinates.

21. \((-8, -6)\) and \((4, 10)\)

22. \((5, 14)\) and \((-3, -9)\)

23. Reason quantitatively. Explain how you know that your answer to Item 22 is reasonable. Remember to use complete sentences and words such as and, or, since, for example, therefore, because of, by the, to make connections between your thoughts.

24. Use your Venn diagram from Item 18 to write a RAFT.
   
   **Role:** Teacher
   
   **Audience:** A classmate who was absent for the lesson on distance between two points
   
   **Format:** Personal note
   
   **Topic:** Explain the mathematical similarities and differences between the Pythagorean Theorem and the Distance Formula.

25. The vertices of \(\triangle XYZ\) are \(X(-3, -6)\), \(Y(21, -6)\), and \(Z(21, 4)\). What is the perimeter of the triangle?

26. Use the Distance Formula to show that \(AB \cong CD\).

![Diagram of a coordinate plane with points A, B, C, and D]
My Notes

Learning Targets:
• Use inductive reasoning to determine the Midpoint Formula.
• Use the Midpoint Formula to find the coordinates of the midpoint of a segment on the coordinate plane.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Visualization, Look for a Pattern, Debriefing, Work Backward, Self Revision/Peer Revision, Discussion Groups, Identify a Subtask

In the previous activity, you defined *midpoint* as a point on a segment that divides it into two congruent segments. Follow the steps below to explore the concept of midpoint on the coordinate plane.

![Coordinate Plane Diagram]

1. In the table below, write the coordinates of the endpoints of each segment shown on the coordinate plane.

   **Use appropriate tools strategically.** Use a ruler to help you identify the midpoint of each segment. Then write the coordinates of the midpoint in the table.

<table>
<thead>
<tr>
<th>Segment</th>
<th>Endpoints</th>
<th>Midpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{AB} )</td>
<td>( A(_, _) ) and ( B(_, _) )</td>
<td>( (_, _) )</td>
</tr>
<tr>
<td>( \overline{CD} )</td>
<td>( C(_, _) ) and ( D(_, _) )</td>
<td>( (_, _) )</td>
</tr>
<tr>
<td>( \overline{EF} )</td>
<td>( E(_, _) ) and ( F(_, _) )</td>
<td>( (_, _) )</td>
</tr>
<tr>
<td>( \overline{GH} )</td>
<td>( G(_, _) ) and ( H(_, _) )</td>
<td>( (_, _) )</td>
</tr>
</tbody>
</table>

2. Use the coordinates in the table in Item 1.
   a. Compare the \( x \)-coordinates of the endpoints of each segment with the \( x \)-coordinate of the midpoint of the segment. Describe the pattern you see.

MATH TIP
For a segment on a number line, the coordinate of the midpoint is the average of the coordinates of the endpoints.
b. Make use of structure. Compare the $y$-coordinates of the endpoints of each segment with the $y$-coordinate of the midpoint of the segment. Describe the pattern you see.

3. Use the patterns you described in Item 2 to write a formula for the coordinates of the midpoint $M$ of a line segment with endpoints at $(x_1, y_1)$ and $(x_2, y_2)$.

Look back at the chart in Item 1 to verify the formula you wrote.

4. Use your formula to find the coordinates of the midpoint $M$ of $\overline{AC}$ with endpoints $A(1, 4)$ and $C(11, 10)$. What can you conclude about segments $AM$ and $MC$?

5. Explain how you could check that your answer to Item 4 is reasonable.

6. Suppose a segment on the coordinate plane is vertical. Can you use the Midpoint Formula to find the coordinates of its midpoint? Explain.

7. The midpoint $M$ of $\overline{ST}$ has coordinates (3, 6). Point $S$ has coordinates (1, 2). What are the coordinates of point $T$? Explain how you determined your answer.

8. Reason quantitatively. The origin, $(0, 0)$, is the midpoint of a segment. What conclusions can you draw about the coordinates of the endpoints of the segment?
LESSON 5-2 PRACTICE

Find the coordinates of the midpoint of each segment with the given endpoints.

9. \( Q(−3, 14) \) and \( R(7, 5) \)

10. \( S(13, 7) \) and \( T(−2, −7) \)

11. \( E(4, 11) \) and \( F(−11, −5) \)

12. \( A(−5, 4) \) and \( B(−5, 18) \)

13. Find and explain the errors that were made in the following calculation of the coordinates of a midpoint. Then fix the errors and determine the correct answer.

Find the coordinates of the midpoint \( M \) of the segment with endpoints \( R(−2, 3) \) and \( S(13, −7) \).

\[
M = \left(\frac{-2 + 3}{2}, \frac{13 + (-7)}{2}\right)
\]

\[
= \left(\frac{1}{2}, \frac{6}{2}\right) = \left(\frac{1}{2}, 3\right)
\]

14. Make sense of problems. \( \overline{HJ} \) is graphed on a coordinate plane. Explain how you would determine the coordinates of the point on the segment that is \( \frac{1}{4} \) of the distance from \( H \) to \( J \).
ACTIVITY 5 PRACTICE
Write your answers on notebook paper.
Show your work.

Lesson 5-1
1. Calculate the distance between the points $A(-4, 2)$ and $B(15, 6)$.
2. Calculate the distance between the points $R(1.5, 7)$ and $S(-2.3, -8)$.
3. Describe how to find the distance between two points on the coordinate plane.
4. To the nearest unit, what is $FG$?

For Items 9–11, determine the length of each segment with the given endpoints.

9. $C(1, 4)$ and $D(11, 28)$
10. $Y(-2, 6)$ and $Z(5, -8)$
11. $P(-7, -7)$ and $Q(9, 5)$
12. Point $R$ has coordinates $(1, 3)$, and point $S$ has coordinates $(6, y)$. If the distance from $R$ to $S$ is 13 units, what are the possible values of $y$?
13. Use the Distance Formula to show that $\triangle ABC$ is isosceles.

8. On a map, a trailhead is located at $(5, 3)$ and a turnaround point is located at $(15, 6)$. Each unit on the map represents 1 km. Ana and Larissa start at the turnaround point and walk directly toward the trailhead. If they walk at an average speed of 4.5 km/h, will they make it to the trailhead in less than 2 hours? Support your answer.

For Items 9–11, determine the length of each segment with the given endpoints.
Lesson 5-2

15. Determine the coordinates of the midpoint of the segment with endpoints \( R(3, 16) \) and \( S(7, -6) \).

16. Determine the coordinates of the midpoint of the segment with endpoints \( W(-5, 10.2) \) and \( X(12, 4.5) \).

17. Point \( C \) is the midpoint of \( AB \). Point \( A \) has coordinates \((2, 4)\), and point \( C \) has coordinates \((5, 0)\).
   a. What are the coordinates of point \( B \)?
   b. What is \( AB \)?
   c. What is \( BC \)?

18. \( JL \) has endpoints \( J(8, 10) \) and \( L(20, 5) \). Point \( K \) has coordinates \((13, 9)\).
   a. Is point \( K \) the midpoint of \( JL \)? Explain how you know.
   b. How could you check that your answer to part a is reasonable?

19. Find the coordinates of the midpoint of \( DE \).

20. What are the coordinates of the midpoint of the segment with endpoints at \((-3, -4)\) and \((5, 8)\)?
   a. \((1, 2)\)
   b. \((2, 4)\)
   c. \((4, 6)\)
   d. \((8, 12)\)

21. A circle on the coordinate plane has a diameter with endpoints at \((6, 8)\) and \((15, 8)\).
   a. What are the coordinates of the center of the circle?
   b. What is the diameter of the circle?
   c. What is the radius of the circle?
   d. Identify the coordinates of another point on the circle. Explain how you found your answer.

22. Two explorers on an expedition to the Arctic Circle have radioed their coordinates to base camp. Explorer A is at coordinates \((-26, -15)\). Explorer B is at coordinates \((13, 21)\). The base camp is located at the origin.
   a. Determine the linear distance between the two explorers.
   b. Determine the midpoint between the two explorers.
   c. Determine the distance between the midpoint of the explorers and the base camp.

23. A segment has endpoints with coordinates \((a, 2b)\) and \((-3a, 4b)\). Write the coordinates of the midpoint of the segment in terms of \( a \) and \( b \).

24. Point \( J \) is the midpoint of \( FG \) with endpoints \( F(1, 4) \) and \( G(5, 12) \). Point \( K \) is the midpoint of \( GH \) with endpoints \( G(5, 12) \) and \( H(-1, 4) \). What is \( JK \)?

MATHEMATICAL PRACTICES
Look for and Express Regularity in Repeated Reasoning

25. Let \((x_1, y)\) and \((x_2, y)\) represent the coordinates of the endpoints of a horizontal segment.
   a. Write and simplify a formula for the distance \( d \) between the endpoints of a horizontal segment.
   b. Write and simplify a formula for the coordinates of the midpoint \( M \) of a horizontal segment.
Each year, a charity hosts a walk as a fundraising event. The walk starts in Reese Park and then continues along Bell Avenue.

1. The route of the first section of the walk is along an east-west path in the park. The walk starts at point $A$, at marker 32 on the path, and continues to point $B$, at marker 4, as shown in the diagram.

   a. The markers on the path are spaced 200 meters apart. What is the distance in meters between points $A$ and $B$?

   b. A water station is located at the midpoint of $AB$. At what marker on the path is the water station located?

   c. A first-aid booth is located $\frac{3}{4}$ of the distance from point $A$ to point $B$. At what marker on the path is the first-aid booth located?

2. The second section of the walk continues from point $B$ along Bell Avenue to the finish line at point $C$, as shown on the map.

   a. Each unit on the map represents 1 kilometer. What is the distance in kilometers from point $B$ to point $C$?

   b. What is the total length of the race (from point $A$ to point $B$ and from point $B$ to point $C$) in kilometers?

   c. A water station is located at the midpoint $M$ of $BC$. Another water station is located at the midpoint $N$ of $MC$. A race coordinator says that the map coordinates of point $N$ are $(6, 8)$. Is the race coordinator correct? Support your answer.
3. An artist is working on the design of a logo for the T-shirts that the participants in the walk will receive. He starts by drawing $\triangle ADC$ so that it measures $60^\circ$. Next, he draws $DB$ so that $m\angle BDC$ is $10^\circ$ greater than $m\angle ADB$.

a. What is $m\angle ADB$?

b. What is $m\angle BDC$?

c. What postulate did you make use of to answer parts a and b? How did this postulate help you write an equation that could be solved for $x$?

<table>
<thead>
<tr>
<th>Scoring Guide</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Knowledge and Thinking (Items 1b, 2c, 3)</td>
<td>• Clear and accurate understanding of midpoint and the Angle Addition Postulate</td>
<td>• A functional understanding of midpoint and the Angle Addition Postulate</td>
<td>• Partial understanding of midpoint and the Angle Addition Postulate</td>
<td>• Little or no understanding of midpoint and the Angle Addition Postulate</td>
</tr>
<tr>
<td>Problem Solving (Items 1, 2, 3)</td>
<td>• An appropriate and efficient strategy that results in a correct answer</td>
<td>• A strategy that may include unnecessary steps but results in a correct answer</td>
<td>• A strategy that results in some incorrect answers</td>
<td>• No clear strategy when solving problems</td>
</tr>
<tr>
<td>Mathematical Modeling / Representations (Item 3)</td>
<td>• Clear and accurate understanding of creating an equation to represent the Angle Addition Postulate</td>
<td>• A functional understanding of creating an equation to represent the Angle Addition Postulate</td>
<td>• Partial understanding of creating an equation to represent the Angle Addition Postulate</td>
<td>• Inaccurate or incomplete understanding of creating an equation to represent the Angle Addition Postulate</td>
</tr>
</tbody>
</table>
| Reasoning and Communication (Items 2c, 3c) | • Precise use of the term midpoint and other math terms to justify a statement concluding that the given coordinates for point $N$ are incorrect
• Precise explanation of the Angle Addition Postulate and its relationship to $\triangle ADB$, $\triangle BDC$, and $\triangle ADC$ | • Adequate understanding of the term midpoint with somewhat correct explanation to justify a statement concluding that the given coordinates for point $N$ are incorrect
• Adequate explanation of the Angle Addition Postulate and its relationship to $\triangle ADB$, $\triangle BDC$, and $\triangle ADC$ | • Misleading or confusing explanation to justify a statement concluding that the given coordinates for point $N$ are incorrect
• Misleading or confusing explanation of the Angle Addition Postulate and its relationship to $\triangle ADB$, $\triangle BDC$, and $\triangle ADC$ | • Incomplete or inaccurate statement to justify whether or not the given coordinates for point $N$ are correct
• Incomplete or inaccurate explanation of the Angle Addition Postulate and its relationship to $\triangle ADB$, $\triangle BDC$, and $\triangle ADC$ |
Learning Targets:
- Use definitions, properties, postulates, and theorems to justify statements.
- Write two-column proofs to prove theorems about lines and angles.

SUGGESTED LEARNING STRATEGIES: Close Reading, Activating Prior Knowledge, Think-Pair-Share, Discussion Groups

A proof is an argument, a justification, or a reason that something is true. A proof is an answer to the question “why?” when the person asking wants an argument that is indisputable.

There are three basic requirements for constructing a good proof.
- Awareness and knowledge of the definitions of the terms related to what you are trying to prove.
- Knowledge and understanding of postulates and previous proven theorems related to what you are trying to prove.
- Knowledge of the basic rules of logic.

To write a proof, you must be able to justify statements. The statements in Example A are based on the diagram to the right in which lines $AC$, $EG$, and $DF$ all intersect at point $B$. Each of the statements is justified using a property, postulate, or definition.

Example A
Name the property, postulate, or definition that justifies each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. If $\angle ABE$ is a right angle, then $m\angle ABE = 90^\circ$.</td>
<td>Definition of right angle</td>
</tr>
<tr>
<td>b. If $\angle 2 \cong \angle 1$ and $\angle 1 \cong \angle 5$, then $\angle 2 \cong \angle 5$.</td>
<td>Transitive Property</td>
</tr>
<tr>
<td>c. Given: $B$ is the midpoint of $AC$. Prove: $AB \cong BC$</td>
<td>Definition of midpoint</td>
</tr>
<tr>
<td>d. $m\angle 2 + m\angle ABE = m\angle DBE$</td>
<td>Angle Addition Postulate</td>
</tr>
<tr>
<td>e. If $\angle 1$ is supplementary to $\angle FBG$, then $m\angle 1 + m\angle FBG = 180^\circ$.</td>
<td>Definition of supplementary angles</td>
</tr>
</tbody>
</table>

MATH TIP
The Reflexive, Symmetric, and Transitive Properties apply to congruence as well as to equality.
Try These A
Using the diagram from the previous page, reproduced here, name the property, postulate, or definition that justifies each statement.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $EB + BG = EG$</td>
<td></td>
</tr>
<tr>
<td>b. If $\angle 5 \cong \angle 6$, then $BF$ bisects $\angle EBC$.</td>
<td></td>
</tr>
<tr>
<td>c. If $m\angle 1 + m\angle 6 = 90^\circ$, then $\angle 1$ is complementary to $\angle 6$.</td>
<td></td>
</tr>
<tr>
<td>d. If $m\angle 1 + m\angle 5 = m\angle 6 + m\angle 5$, then $m\angle 1 = m\angle 6$.</td>
<td></td>
</tr>
<tr>
<td>e. Given: $\overline{AC} \perp \overline{EG}$ Prove: $\angle ABG$ is a right angle.</td>
<td></td>
</tr>
</tbody>
</table>
Check Your Understanding

1. Explain why the following statement does not need to be justified.

   The midpoint of a segment is a point on the segment that divides it into two congruent segments.

2. Given: $RS$ and $ST$ share endpoint $S$.
   
   **Critique the reasoning of others.** Based on this information, Michaela says that the Segment Addition Postulate justifies the statement that $RS + ST = RT$. Is there a flaw in Michaela’s reasoning, or is she correct? Explain.

3. **Construct viable arguments.** Write and justify two statements based on the information in the figure.

   ![Figure](image)

**LESSON 6-1 PRACTICE**

Lines $CF$, $DH$, and $EA$ intersect at point $B$. Use this figure for Items 4–8. Write the definition, postulate, or property that justifies each statement.

4. If $\angle 2$ is supplementary to $\angle CBE$, then $m\angle 2 + m\angle CBE = 180^\circ$.

5. If $\angle 2 \cong \angle 3$, then $BF$ bisects $\angle GBE$.

6. $CB + BF = CF$

7. If $\angle DBF$ is a right angle, then $HD \perp CF$.

8. If $m\angle 3 = m\angle 6$, then $m\angle 3 + m\angle 2 = m\angle 6 + m\angle 2$.

9. **Reason abstractly.** Write a statement related to the figure above that can be justified by the Angle Addition Postulate.

**MATH TIP**

Do not assume that an angle is a right angle just because it appears to measure $90^\circ$. You can only conclude that an angle is a right angle if (1) you are given this information, (2) a diagram of the angle includes a right angle symbol, or (3) you prove that the angle is a right angle.
Learning Targets:
• Complete two-column proofs to prove theorems about segments.
• Complete two-column proofs to prove theorems about angles.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Think-Pair-Share, Close Reading, Discussion Groups, Self Revision/Peer Revision, Group Presentation

Earlier, you wrote two-column proofs to solve algebraic equations. You justified each statement in these proofs by using an algebraic property. Now you will use two-column proofs to prove geometric theorems. You must justify each statement by using a definition, a postulate, a property, or a previously proven theorem.

Recall that **vertical angles** are opposite angles formed by a pair of intersecting lines. In the figure below, $\angle 1$ and $\angle 2$ are vertical angles. The following example illustrates how to prove that vertical angles are congruent.

**Example A**

Theorem: Vertical angles are congruent.

Given: $\angle 1$ and $\angle 2$ are vertical angles.

Prove: $\angle 1 \cong \angle 2$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $m\angle 1 + m\angle 3 = 180^\circ$</td>
<td>1. Definition of supplementary angles</td>
</tr>
<tr>
<td>2. $m\angle 2 + m\angle 3 = 180^\circ$</td>
<td>2. Definition of supplementary angles</td>
</tr>
<tr>
<td>3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$</td>
<td>3. Substitution Property</td>
</tr>
<tr>
<td>4. $m\angle 1 = m\angle 2$</td>
<td>4. Subtraction Property of Equality</td>
</tr>
<tr>
<td>5. $\angle 1 \cong \angle 2$</td>
<td>5. Definition of congruent angles</td>
</tr>
</tbody>
</table>

**Guided Example B**

Supply the missing statements and reasons.

Theorem: All right angles are congruent.

Given: $\angle A$ and $\angle B$ are right angles.

Prove: $\angle A \cong \angle B$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $m\angle A = 90^\circ; m\angle B = 90^\circ$</td>
<td>2. Definition of</td>
</tr>
<tr>
<td>3. $m\angle A = \angle B$</td>
<td>3. Property</td>
</tr>
<tr>
<td>4. $\angle A \cong \angle B$</td>
<td>4. Definition of</td>
</tr>
</tbody>
</table>
Lesson 6-2
Two-Column Geometric Proofs

Try These A-B

a. Complete the proof.

Given: Q is the midpoint of PR.
QR ≅ RS

Prove: PQ ≅ RS

Statements | Reasons
--- | ---
1. Q is the midpoint of PR. | 1. 
2. PQ ≅ QR | 2. 
3. QR ≅ RS | 3. 
4. PQ ≅ RS | 4.

b. Complete the proof.

Given: ∠1 and ∠2 are supplementary. m∠1 = 68°

Prove: m∠2 = 112°

Statements | Reasons
--- | ---
1. | 1. Given
2. | 2. Definition of supplementary angles
3. | 3. Given
4. | 4. Substitution Property
5. | 5. Subtraction Property of Equality

MATH TIP

More than one statement in a two-column proof can be given information.
Example C

Arrange the statements and reasons below in a logical order to complete the proof.

Theorem: If two angles are complementary to the same angle, then the two angles are congruent.

Given: ∠A and ∠B are each complementary to ∠C.

Prove: ∠A ≅ ∠B

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ∠A and ∠B are each complementary to ∠C.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. m∠A + m∠C = 90°; m∠B + m∠C = 90°</td>
<td>2. Definition of complementary angles</td>
</tr>
<tr>
<td>3. m∠A + m∠C = m∠B + m∠C</td>
<td>3. Transitive Property</td>
</tr>
<tr>
<td>4. m∠A = m∠B</td>
<td>4. Subtraction Property of Equality</td>
</tr>
<tr>
<td>5. ∠A ≅ ∠B</td>
<td>5. Definition of congruent angles</td>
</tr>
</tbody>
</table>

Start the proof with the given information. Then decide which statement and reason follow logically from the first statement. Continue until you have proved that ∠A ≅ ∠B.
Lesson 6-2
Two-Column Geometric Proofs

Try These C

a. Attend to precision. Arrange the statements and reasons below in a logical order to complete the proof.

Given: \( \angle 1 \) and \( \angle 2 \) are vertical angles; \( \angle 1 \cong \angle 3 \).

Prove: \( \angle 2 \cong \angle 3 \)

\[ \angle 1 \cong \angle 2 \quad \text{Vertical angles are congruent} \]

\[ \angle 2 \cong \angle 3 \quad \text{Transitive Property} \]

\[ \angle 1 \cong \angle 3 \quad \text{Given} \]

\[ \angle 1 \text{ and } \angle 2 \text{ are vertical angles.} \quad \text{Given} \]

b. Write a two-column proof of the following theorem.

Theorem: If two angles are supplementary to the same angle, then the two angles are congruent.

Given: \( \angle R \) and \( \angle S \) are each supplementary to \( \angle T \).

Prove: \( \angle R \cong \angle S \)

Check Your Understanding

1. If you know that \( \angle D \) and \( \angle F \) are both complementary to \( \angle I \), what statement could you prove using the Congruent Complements Theorem?

2. What types of information can you list as reasons in a two-column geometric proof?

3. Kenneth completed this two-column proof. What mistake did he make? How could you correct the mistake?

Given: \( JK \) bisects \( \angle KJM \); \( m\angle KJL = 35^\circ \)

Prove: \( m\angle LJM = 35^\circ \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( JK ) bisects ( \angle KJM ).</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle KJL \cong \angle LJM )</td>
<td>2. Definition of congruent angles</td>
</tr>
<tr>
<td>3. ( m\angle KJL = m\angle LJM )</td>
<td>3. Definition of angle bisector</td>
</tr>
<tr>
<td>4. ( m\angle KJL = 35^\circ )</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. ( m\angle LJM )</td>
<td>5. Transitive Property</td>
</tr>
</tbody>
</table>
4. Supply the missing statements and reasons.

**Given:** \( \angle 1 \) is complementary to \( \angle 2 \); \( BE \) bisects \( \angle DBC \).

**Prove:** \( \angle 1 \) is complementary to \( \angle 3 \).

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( BE ) bisects ( \angle DBC )</td>
<td>1.</td>
</tr>
<tr>
<td>2. ( \angle 2 \cong \angle 3 )</td>
<td>2.</td>
</tr>
<tr>
<td>3.</td>
<td>3. Definition of congruent angles</td>
</tr>
<tr>
<td>4. ( \angle 1 ) is complementary to ( \angle 2 ).</td>
<td>4.</td>
</tr>
<tr>
<td>5. ( \angle 1 + m\angle 2 = ____ )</td>
<td>5.</td>
</tr>
<tr>
<td>6. ( m\angle 1 + m\angle 3 = 90^\circ )</td>
<td>6.</td>
</tr>
<tr>
<td>7.</td>
<td>7. Definition of complementary angles</td>
</tr>
</tbody>
</table>

**Construct viable arguments.** Write a two-column proof for each of the following.

5. **Given:** \( M \) is the midpoint of \( LN \); \( LM = 8 \).
   **Prove:** \( LN = 16 \)

6. **Given:** \( BD \) bisects \( \angle ABC \); \( m\angle DBC = 90^\circ \).
   **Prove:** \( \angle ABC \) is a straight angle.

7. **Given:** \( PQ \cong QR \), \( QR = 14 \), \( PR = 14 \)
   **Prove:** \( PQ \cong PR \)

8. **Reason abstractly.** What type of triangle is shown in Item 7? Explain how you know.
ACTIVITY 6 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 6-1

Use this diagram to identify the property, postulate, or theorem that justifies each statement in Items 1–4.

1. \(PQ + QR = PR\)
   A. Angle Addition Postulate
   B. Addition Property
   C. Definition of congruent segments
   D. Segment Addition Postulate

2. If \(Q\) is the midpoint of \(PR\), then \(PQ \cong QR\).
   A. Definition of midpoint
   B. Definition of congruent segments
   C. Definition of segment bisector
   D. Segment Addition Postulate

3. \(\angle 3 \cong \angle 4\)
   A. Definition of supplementary
   B. Definition of congruent angles
   C. Vertical angles are congruent
   D. Definition of angle bisector

4. If \(\angle 1\) is complementary to \(\angle 2\), then \(m\angle 1 + m\angle 2 = 90^\circ\).
   A. Angle Addition Postulate
   B. Addition Property
   C. Definition of perpendicular
   D. Definition of complementary

Lesson 6-2

9. Complete the proof.
   \(\text{Given: } XY = 6, XZ = 14\)
   \(\text{Prove: } YZ = 8\)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (XY = 6, XZ = 14)</td>
<td>1.</td>
</tr>
<tr>
<td>2. (XY + YZ = XZ)</td>
<td>2.</td>
</tr>
<tr>
<td>3. (6 + YZ = 14)</td>
<td>3.</td>
</tr>
<tr>
<td>4. (YZ = 8)</td>
<td>4.</td>
</tr>
</tbody>
</table>

10. Based on the given information in Item 9, can you conclude that \(Y\) is the midpoint of \(XZ\)? Explain your reasoning.
11. Supply the missing statements and reasons.

**Given:** \( \overline{PR} \) bisects \( \angle QPS \); \( \angle QPS \) is a right angle.

**Prove:** \( m\angle RPS = 45^\circ \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{PR} ) bisects ( \angle QPS ).</td>
<td>1. _____</td>
</tr>
<tr>
<td>2. ( m\angle QPR = m\angle RPS )</td>
<td>2. Definition of _____</td>
</tr>
<tr>
<td>3. ( \angle QPS ) is a right angle.</td>
<td>3. _____</td>
</tr>
<tr>
<td>4. ( m\angle QPS = _____ )</td>
<td>4. Definition of _____</td>
</tr>
<tr>
<td>5. ( m\angle QPR + m\angle RPS = m\angle QPS )</td>
<td>5. _____ Postulate</td>
</tr>
<tr>
<td>6. ( m\angle RPS + m\angle RPS = 90^\circ )</td>
<td>6. _____ Property</td>
</tr>
<tr>
<td>7. ( 2(m\angle RPS) = 90^\circ )</td>
<td>7. Distributive Property</td>
</tr>
<tr>
<td>8. ( m\angle RPS = _____ )</td>
<td>8. _____</td>
</tr>
</tbody>
</table>

12. Complete the proof.

**Given:** \( AC = 2(AB) \)

**Prove:** \( B \) is the midpoint of \( AC \).

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2.</td>
<td>2. Segment Addition Postulate</td>
</tr>
<tr>
<td>3.</td>
<td>3. Transitive Property</td>
</tr>
<tr>
<td>4.</td>
<td>4. Subtraction Property of Equality</td>
</tr>
<tr>
<td>5.</td>
<td>5. Definition of congruent segments</td>
</tr>
<tr>
<td>6.</td>
<td>6. Definition of midpoint</td>
</tr>
</tbody>
</table>

13. Arrange the statements and reasons below in a logical order to complete the proof.

**Given:** \( \angle 1 \) and \( \angle 2 \) are complementary; \( m\angle 1 = 28^\circ \).

**Prove:** \( m\angle 2 = 62^\circ \)

<table>
<thead>
<tr>
<th>28° + m\angle 2 = 90°</th>
<th>Substitution Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>m\angle 1 + m\angle 2 = 90°</td>
<td>Definition of complementary angles</td>
</tr>
<tr>
<td>m\angle 1 = 28°</td>
<td>Given</td>
</tr>
<tr>
<td>m\angle 2 = 62°</td>
<td>Subtraction Property of Equality</td>
</tr>
<tr>
<td>( \angle 1 ) and ( \angle 2 ) are complementary.</td>
<td>Given</td>
</tr>
</tbody>
</table>

14. Write a two-column proof.

**Given:** \( \angle 1 \cong \angle 2, m\angle 4 = 30^\circ \)

**Prove:** \( m\angle 2 = 30^\circ \)

15. In Item 14, what are the measures of \( \angle 3, \angle 5, \) and \( \angle 6 \)? Explain how you know.

**MATHEMATICAL PRACTICES**

**Construct Viable Arguments and Critique the Reasoning of Others**

16. Cara says that the following statement can be justified by the definition of vertical angles.

Is her reasoning correct? Explain.

*If \( \angle G \) and \( \angle H \) are vertical angles, then \( \angle G \cong \angle H \).*
Learning Targets:

- Make conjectures about the angles formed by a pair of parallel lines and a transversal.
- Prove theorems about these angles.

**SUGGESTED LEARNING STRATEGIES:** Summarizing, Paraphrasing, Vocabulary Organizer, Interactive Word Wall, Predict and Confirm, Think-Pair-Share, Discussion Groups, Group Presentation, Self Revision/Peer Revision

Matt works for a company called Patios by Madeline. A new customer has asked him to design a patio and walkway. The rows of bricks in the patio will be parallel to the walkway, as shown in the diagram.

So far, Matt has used stakes to tie down two parallel strings that he can use to align rows of bricks. He has also used paint to mark the underground gas line to the house so that he can avoid accidents during construction. The string lines and the gas line intersect to form eight angles.

**MATH TERMS**

Two lines (or parts of lines) are **parallel** if they are coplanar and do not intersect.
1. **Use appropriate tools strategically.** Use a protractor to measure each of the numbered angles formed by the string lines and the gas line on the previous page.

\[
\begin{align*}
\angle 1 &= \angle 2 \\
\angle 3 &= \angle 4 \\
\angle 5 &= \angle 6 \\
\angle 7 &= \angle 8 \\
\end{align*}
\]

2. The gas line is a **transversal** of the string lines. Name the angle pairs formed by these lines that match each description.

   a. Four pairs of **corresponding angles**
   
   b. Two pairs of **same-side interior angles**
   
   c. Two pairs of **alternate interior angles**

3. **Express regularity in repeated reasoning.** Based on your answers to Items 1 and 2, make a conjecture about each type of angle pair formed by parallel lines and a transversal.

   a. Corresponding angles
   
   b. Same-side interior angles
   
   c. Alternate interior angles
4. The lines below are parallel.
   a. Draw a transversal to the parallel lines.
   
   
   b. Number the angles formed above and record the measure of each angle.

5. **Construct viable arguments.** Do your answers to Item 4 **confirm** the conjectures you made in Item 3? Explain. Revise your conjectures if needed.
6. The Corresponding Angles Postulate involves angles formed by two parallel lines cut by a transversal.
   a. **Reason abstractly.** Based on your earlier conjecture, write the Corresponding Angles Postulate in if-then form.

   b. Do you need to prove that this postulate is true before you can use it as a reason in a proof? Explain.

7. The Alternate Interior Angles Theorem and the Same-Side Interior Angles Theorem also involve angles formed by two parallel lines cut by a transversal.
   a. Based on your earlier conjecture, write the Alternate Interior Angles Theorem in if-then form.

   b. Based on your earlier conjecture, write the Same-Side Interior Angles Theorem in if-then form.

**MATH TIP**
You will prove these theorems later in this activity.
Lesson 7-1
Parallel Lines and Angle Relationships

Check Your Understanding

In the diagram, $\ell \parallel m$. Use the diagram for Items 8–10.

8. Explain how you know that $\angle 4$ and $\angle 6$ are same-side interior angles.

9. Are $\angle 2$ and $\angle 3$ corresponding angles? Explain how you know.

10. If $m\angle 5 = 65^\circ$, then what is $m\angle 4$? Support your answer.

11. $\angle BCD$ and $\angle CGE$ are corresponding angles formed by two parallel lines cut by a transversal. Given that $m\angle BCD = 3x + 6$ and $m\angle CGE = x + 24$, complete the following.
   a. What is the value of $x$?
   b. What is $m\angle BCD$?
   c. Explain how you found your answers.

12. Reason quantitatively. Complete the following proof of the Same-Side Interior Angles Theorem.

   Given: $m \parallel n$
   Prove: $\angle 4$ and $\angle 5$ are supplementary.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $m \parallel n$</td>
<td>1.</td>
</tr>
<tr>
<td>2. $\angle 1 \cong \angle 5$</td>
<td>2. Corresponding Angles Postulate</td>
</tr>
<tr>
<td>3. $m\angle 1 = m\angle 5$</td>
<td>3.</td>
</tr>
<tr>
<td>4. $m\angle 1 + m\angle 4 = 180^\circ$</td>
<td>4.</td>
</tr>
<tr>
<td>5. $m\angle 5 + m\angle 4 = 180^\circ$</td>
<td>5.</td>
</tr>
<tr>
<td>6.</td>
<td>6. Definition of Supplementary Angles</td>
</tr>
</tbody>
</table>

MATH TIP

The symbol $\parallel$ means “is parallel to.” So, $\ell \parallel m$ means “line $\ell$ is parallel to line $m$.”

It is sufficient to prove that one pair of same-side interior angles formed by a pair of parallel lines and a transversal are supplementary.
Lesson 7-1
Parallel Lines and Angle Relationships

LESSON 7-1 PRACTICE

In the diagram, \( a \parallel b \). Use the diagram for Items 15–20.
Determine whether each statement in Items 15–18 is true or false.
Justify your response with the appropriate postulate or theorem.

15. \( \angle 2 \) is supplementary to \( \angle 3 \).
16. \( \angle 8 \cong \angle 6 \)
17. \( \angle 7 \) is supplementary to \( \angle 3 \).
18. \( \angle 6 \cong \angle 4 \)
19. If \( m\angle 2 = 8x - 20 \), and \( m\angle 3 = 5x + 5 \),
what is \( m\angle 2 \)? What is \( m\angle 3 \)?
20. Reason quantitatively. Based on your answer to Item 19, what are
the measures of the other numbered angles in the diagram? Explain
your reasoning.
Lesson 7-2
Proving Lines are Parallel

Learning Targets:
• Develop theorems to show that lines are parallel.
• Determine whether lines are parallel.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Predict and Confirm, Think-Pair-Share, Debriefing, Discussion Groups, Group Presentation

1. Make use of structure. State the converse of each postulate or theorem.
   a. Same-Side Interior Angles Theorem

   b. Alternate Interior Angles Theorem

   c. Corresponding Angles Postulate

With your group, reread the problem scenario as needed. Make notes on the information provided in the problem. Respond to questions about the meaning of key information and organize the information needed to create a reasonable solution.

Matt is working on the blueprint for the new patio, as shown on the next page. Add information to the blueprint as you work through this lesson.

To ensure that the rows of bricks will be parallel to the walkway, Matt extends the string line along the edge of the walkway and labels points, X, A, and Y. He also labels points B, C, D, and E where additional string lines will cross the gas line.

2. Follow these steps.
   a. Use a protractor to measure ∠HAX.

   b. Locate a stake on the left edge of the patio at point P by drawing BP so that m∠HBP = m∠HAX.

   c. Locate a stake on the right edge of the patio at point J by extending BP in the opposite direction.

MATH TIP
The converse of the statement “If p, then q” is “If q, then p.” If a conditional statement is true, the converse of the statement may or may not be true.
3. For which converse from Item 1 does your drawing from Item 2 provide support? Explain.

You drew $\angle HBP$ and $\angle HAX$ so that they have the same measure. Identify the relationship between these two angles to help you answer Item 3.
4. Matt realizes that he is not limited to using corresponding angles to draw parallel lines. Use a protractor to draw another parallel string through point $C$ on the blueprint, using alternate interior angles. Mark the angles that you used to draw this new parallel line.

5. Explain how your drawing from Item 4 provides support for the converse of the Alternate Interior Angles Theorem.

6. **Use appropriate tools strategically.** Use a protractor to draw another parallel string through point $E$ on the blueprint, using same-side interior angles. Mark the angles that were used to draw this new parallel line.

7. **Reason abstractly.** Explain how your drawing from Item 6 provides support for the converse of the Same-Side Interior Angles Theorem.
The converses of the Same-Side Interior Angles Theorem, the Alternate Interior Angles Theorem, and the Corresponding Angles Postulate are all true, as indicated by the string lines you drew on the blueprint.

Look back at Item 1 in this lesson to review the converses of these theorems and postulate.

**8. Attend to precision.** Matt’s assistant sets up two string lines as shown. Matt finds that \( m \angle 2 = 63^\circ \) and \( m \angle 6 = 65^\circ \). Explain how Matt can tell that the strings are not parallel.
Lesson 7-2
Proving Lines are Parallel

Check Your Understanding

9. Refer back to Item 8. How could Matt adjust the strings so that they are parallel?

10. What is the inverse of the Alternate Interior Angles Theorem? Is the inverse true? Explain how you know.

11. Complete the following proof of the Converse of the Corresponding Angles Postulate.
   Given: \( \angle 1 \cong \angle 5 \)
   Prove: \( m \parallel n \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
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<tbody>
<tr>
<td>1.</td>
<td>1.</td>
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<tr>
<td>2. ( m\angle 1 = m\angle 5 )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( m\angle 1 + m\angle 4 = 180^\circ )</td>
<td>3. Linear Pair Postulate</td>
</tr>
<tr>
<td>4. ( m\angle 5 + m\angle 4 = 180^\circ )</td>
<td>4.</td>
</tr>
<tr>
<td>5. ( \angle 5 ) and ( \angle 4 ) are supplementary.</td>
<td>5.</td>
</tr>
<tr>
<td>6.</td>
<td>6. Converse of the Same-Side Interior Angles Theorem</td>
</tr>
</tbody>
</table>

LESSON 7-2 PRACTICE

For Items 12–14, use the diagram to answer each question. Then justify your answer.

12. Given that \( m\angle 1 = 52^\circ \) and \( m\angle 7 = 52^\circ \), is \( m \parallel p \)?

13. Given that \( m\angle 11 = 52^\circ \) and \( m\angle 3 = 56^\circ \), is \( m \parallel n \)?

14. Given that \( m\angle 10 = 124^\circ \) and \( m\angle 7 = 52^\circ \), is \( n \parallel p \)?

15. Reason abstractly. Two lines are cut by a transversal such that a pair of alternate interior angles are right angles. Are the two lines parallel? Explain.

16. Model with mathematics. Describe how a stadium worker can determine whether two yard lines painted on a football field are parallel. Assume that the worker has a protractor, string, and two stakes.
**Learning Targets:**
- Develop theorems to show that lines are perpendicular.
- Determine whether lines are perpendicular.

**SUGGESTED LEARNING STRATEGIES:** Activating Prior Knowledge, Think Aloud, Think-Pair-Share, Create Representations, RAFT

A second customer of Patios by Madeline has hired the company to build a patio with rows of bricks that are *perpendicular* to the walkway, as shown at right.

Matt is planning the design for this patio. So far, he has extended a line along the walkway and labeled points X and Y to mark stakes at the edge of the patio. He has also drawn point W to mark another stake at the edge of the patio, as shown below.

1. Use a protractor to draw the line perpendicular to $XY$ that passes through W. Label the point at which the line intersects $XY$ point Z.

2. **Use appropriate tools strategically.** Use your protractor and your knowledge of parallel lines to draw a line, across the patio, parallel to WZ. Explain how you know the lines are parallel.
3. Describe the relationship between the newly drawn line and \( XY \).

4. Critique the reasoning of others. Matt reasons that if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line. Show that Matt’s conjecture is true. Include a diagram in your answer.

5. Matt draws \( AB \) on his design of the patio so that \( XY \) is the perpendicular bisector of \( AB \). \( XY \) intersects \( AB \) at point \( M \). List three conclusions you can draw from this information.

**MATH TERMS**

The **Perpendicular Transversal Theorem** states that if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.

**MATH TIP**

In Item 4, start by drawing line \( m \) parallel to line \( n \). Then draw transversal \( t \) so that it is perpendicular to \( m \).

You want to show that \( t \perp n \) given that \( m \parallel n \) and \( t \perp m \). Number the angles in your diagram as needed so that you can refer to them in your answer.

**MATH TERMS**

A **perpendicular bisector** of a segment is a line (or part of a line) that intersects the segment at its midpoint to form right angles.
6. Matt’s boss Madeline is so impressed with Matt’s work that she asks him to write an instruction guide for creating rows of bricks parallel to a patio walkway. Madeline asks that the guide provide enough information so that a bricklayer can use any pair of angles (same-side interior, alternate interior, or corresponding) to determine the location of each pair of stakes. She also requests that Matt include directions for creating rows of bricks that are perpendicular to a walkway. Write Matt’s instruction guide, following Madeline’s directions.

7. \textbf{Reason abstractly.} Lines \( a, b, \) and \( c \) lie in the same plane. If \( a \perp b \) and \( b \perp c \), can you conclude that \( a \perp c \)? Explain. Include a drawing to support your answer.

8. \( RS \) is the perpendicular bisector of \( JL \). Can you conclude that point \( K \) is the midpoint of \( RS \)? Explain.

\textbf{Check Your Understanding}

\textbf{LESSON 7-3 PRACTICE}

In the diagram, \( \ell \parallel m \), \( m\angle 1 = 90^\circ \), and \( \angle 5 \) is a right angle. Use the diagram for Items 9–11.

9. Explain how you know that \( \ell \perp p \).
10. Show that \( m\angle 4 = 90^\circ \).
11. \textbf{Make use of structure.} Show that \( \ell \parallel n \).
12. The perpendicular bisector of \( CD \) intersects \( CD \) at point \( P \). If \( CP = 12 \), what is \( CD \)?
13. An angle formed by the intersection of two lines is obtuse. Could the lines be perpendicular? Explain.
ACTIVITY 7 PRACTICE
Write your answers on notebook paper. Show your work.

Lesson 7-1
Use this diagram for Items 1–10.

1. List all angles in the diagram that form a corresponding angle pair with \( \angle 5 \).
2. List all angles in the diagram that form an alternate interior angle pair with \( \angle 14 \).
3. List all angles in the diagram that form a same-side interior angle pair with \( \angle 14 \).
4. Given \( \ell \parallel n \). If \( m \angle 6 = 120^\circ \), then \( m \angle 16 = ? \).
5. Given \( \ell \parallel n \). If \( m \angle 12 = 5x + 10 \) and \( m \angle 14 = 6x - 4 \), then \( x = ? \) and \( m \angle 4 = ? \).
6. Given \( \ell \parallel n \). If \( m \angle 14 = 75^\circ \), then \( m \angle 2 = ? \).
7. Given \( \ell \parallel n \). If \( m \angle 9 = 3x + 12 \) and \( m \angle 15 = 2x + 27 \), then \( x = ? \) and \( m \angle 10 = ? \).
8. Given \( \ell \parallel n \). If \( m \angle 3 = 100^\circ \), then \( m \angle 2 = ? \).
9. Given \( \ell \parallel n \). If \( m \angle 16 = 5x + 18 \) and \( m \angle 15 = 3x + 2 \), then \( x = ? \) and \( m \angle 8 = ? \).
10. If \( m \angle 10 = 135^\circ \), \( m \angle 12 = 75^\circ \), and \( \ell \parallel n \) determine the measure of each angle. Justify each answer.
   a. \( m \angle 16 \)
   b. \( m \angle 15 \)
   c. \( m \angle 8 \)
   d. \( m \angle 14 \)
   e. \( m \angle 3 \)

11. Supply the missing statements and reasons.

Given: \( \ell \parallel m \)
Prove: \( \angle 1 \cong \angle 7 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 1 \cong \angle 5 )</td>
<td>2.</td>
</tr>
<tr>
<td>3. ( \angle 5 \cong \angle 7 )</td>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
<td>4.</td>
</tr>
</tbody>
</table>

12. Two parallel lines are cut by a transversal. How are the corresponding angles and same-side interior angles alike?
   A. The angles in each pair are congruent.
   B. The angles in each pair are supplementary.
   C. The angles in each pair are between the parallel lines.
   D. The angles in each pair are on the same side of the transversal.

Lesson 7-2
Use this diagram for Items 13–15.

13. If \( m \angle 2 = 110^\circ \) and \( m \angle 3 = 80^\circ \), is \( f \parallel g \)? Justify your answer.
14. If \( \angle 8 \cong \angle 4 \), is \( f \parallel g \)? Justify your answer.
15. Given that \( m \angle 5 = 12x + 4 \) and \( m \angle 7 = 10x + 16 \), what must the value of \( x \) be in order for line \( f \) to be parallel to line \( g \)?
16. If $a \parallel b$ and $b \parallel c$, can you conclude that $a \parallel c$? Explain. Include a drawing to support your answer.

17. Arrange the statements and reasons below in a logical order to complete the proof of the Converse of the Alternate Interior Angles Theorem.

Given: $\angle 4 \cong \angle 6$

Prove: $m \parallel n$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m\angle 5 + m\angle 6 = 180^\circ$</td>
<td>Linear Pair Postulate</td>
</tr>
<tr>
<td>$\angle 4 \cong \angle 6$</td>
<td>Given</td>
</tr>
<tr>
<td>$m\angle 5 + m\angle 4 = 180^\circ$</td>
<td>Substitution Property</td>
</tr>
<tr>
<td>$\angle 5$ and $\angle 4$ are supplementary.</td>
<td>Definition of supplementary angles</td>
</tr>
<tr>
<td>$m\angle 4 = m\angle 6$</td>
<td>Definition of congruent angles</td>
</tr>
<tr>
<td>$m \parallel n$</td>
<td>Converse of the Same-Side Interior Angles Theorem</td>
</tr>
</tbody>
</table>

Lesson 7-3

Use this diagram for Items 18–20.

18. What information do you need to know to prove that line $m$ is the perpendicular bisector of $AC$?

19. Given that line $m$ is the perpendicular bisector of $AC$, $AB = 4x - 1$, and $BC = 2x + 7$, what is the value of $x$?

20. Based on your answer to Item 19, what is $AC$?
   - A. 11
   - B. 15
   - C. 22
   - D. 30

21. How many lines perpendicular to line $l$ can you draw through point $P$? Explain how you know.

22. Complete the proof.

   **Theorem:** If two lines in the same plane are perpendicular to the same line, then the lines are parallel to each other.

   Given: $\ell \perp n$, $m \perp n$

   Prove: $\ell \parallel m$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 2$ is a right angle; $\angle 6$ is a right angle</td>
<td>2.</td>
</tr>
<tr>
<td>3. $\angle 2 \cong \angle 6$</td>
<td>3.</td>
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<tr>
<td>4.</td>
<td>4.</td>
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</tbody>
</table>

**MATHEMATICAL PRACTICES**

Look for and Express Regularity in Repeated Reasoning

23. Suppose you are given the measure of one of the angles formed when a pair of parallel lines is cut by a transversal. Explain how to use this measure to find the measures of the other seven angles.
The new ramp at the local skate park is shown above. In addition to the wooden ramp, an aluminum rail (not shown) is mounted to the edge of the ramp. While the image of the ramp may conjure thoughts of kickflips, nollies, and nose grinds, there are mathematical forces at work here as well.

1. Use the diagram of the ramp to complete the chart below:

<table>
<thead>
<tr>
<th>Describe two parts of the ramp that appear to be parallel.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Describe two parts of the ramp that appear to be perpendicular.</td>
</tr>
<tr>
<td>Describe two parts of the ramp that appear to be neither parallel nor perpendicular.</td>
</tr>
</tbody>
</table>
2. Recall that the slope of a line is the ratio of the vertical change to the horizontal change between two points on the line. If \((x_1, y_1)\) and \((x_2, y_2)\) are two points on a line and the slope is given by \(m\), explain why the slope can be calculated using the equation \[ m = \frac{y_2 - y_1}{x_2 - x_1}. \]

3. The diagram above shows a cross-section of the skate ramp and its railing transposed onto a coordinate grid.
   a. Calculate the slopes of \(XW\) and \(XU\).
   b. Explain why the slopes of \(XW\) and \(XU\) are equivalent to the slope of \(XT\).

4. Find the slopes of the following segments.
   a. \(AB\)
   b. \(XT\)
   c. \(WG\)
   d. \(TJ\)
   e. \(AD\)

5. Model with mathematics. Complete the chart by stating whether each pair of segments appear to be parallel, perpendicular, or neither. Then list the slopes of the segments.
Lesson 8-1
Slopes of Parallel and Perpendicular Lines

<table>
<thead>
<tr>
<th>Segments</th>
<th>Parallel, Perpendicular, or Neither?</th>
<th>Slopes</th>
</tr>
</thead>
<tbody>
<tr>
<td>( AB ) and ( XT )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( WG ) and ( TJ )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( XT ) and ( WG )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( XT ) and ( TJ )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( AB ) and ( AD )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( WG ) and ( AD )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6. Express regularity in repeated reasoning. Based on the chart in Item 5, make conjectures about the slopes of each type of segments.

a. Parallel segments

b. Perpendicular segments

c. Segments that are neither parallel nor perpendicular

Check Your Understanding

7. Do the conjectures you wrote in Item 6 apply to lines as well as segments? Explain.

8. Reason abstractly. Are all horizontal lines parallel? Use slope to explain how you know.

9. \( JK \) has a slope of \( \frac{2}{5} \), and \( MN \) has a slope of \( \frac{5}{2} \). Are these segments parallel, perpendicular, or neither? Explain.

LESSON 8-1 PRACTICE

Use slope to support your answers to Items 10–12.

10. Is \( AB \parallel DC \)?

11. Is \( AB \perp AD \)?

12. Is \( DC \perp AD \)?

13. Reason quantitatively. Line \( \ell \) passes through the origin and the point (3, 4). What is the slope of a line parallel to line \( \ell \)?

14. \( \angle RST \) is a right angle. If \( RS \) has a slope of 3, what must be the slope of \( ST \)? Explain.
Learning Targets:

• Write the equation of a line that is parallel to a given line.
• Write the equation of a line that is perpendicular to a given line.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Look for a Pattern, Identify a Subtask, Discussion Groups, Create Representations, Think-Pair-Share, Self Revision/Peer Revision, Visualization, Identify a Subtask

Here again is the diagram of the skate ramp and its railing.

1. The equation of the line containing $BD$ is $y = -\frac{2}{3} x + 4$. Identify the slope and the $y$-intercept of this line.

2. Based on your conjectures in Item 4 in the previous lesson, what would be the slope of the line containing $JM$? Use the formula for slope to verify your answer or provide an explanation of the method you used to determine the slope.

3. Identify the $y$-intercept of the line containing $JM$. Use the slope and the $y$-intercept to write the equation of this line.

4. If $JM \perp RJ$, identify the slope, $y$-intercept, and the equation for the line containing $RJ$.

The skate park will also have an angled grind rail. A drafter is drawing the grind rail on a coordinate plane.

6. **Make sense of problems.** The drafter has already drawn line $n$. Next, the drafter needs to draw line $p$ so that it is parallel to line $n$ and passes through point $(-2, 3)$.
   a. What is the slope of line $p$? How do you know?

   b. What is the equation of line $p$ written in point-slope form?

   c. What is the equation of line $p$ written in slope-intercept form?

   d. Sketch line $p$ on the drafter’s coordinate plane.
7. Next, the drafter needs to draw line $q$ so that it is perpendicular to line $n$ and passes through point $(-9, 7)$.

a. What is the slope of line $q$? How do you know?

b. What is the equation of line $q$ written in point-slope form?

c. What is the equation of line $q$ written in slope-intercept form?

d. Sketch line $q$ on the drafter's coordinate plane.

8. **Model with mathematics.** Finally, the drafter needs to draw line $r$ so that it is perpendicular to line $n$ and passes through point $(8, 0)$.

a. What is the equation of line $r$ written in point-slope form?

b. What is the equation of line $r$ written in slope-intercept form?

c. Sketch line $r$ on the drafter's coordinate plane.
9. **Reason abstractly and quantitatively.** Describe the relationships among the lines $n$, $p$, $q$, and $r$.

Quadrilaterals can be classified by the presence or absence of parallel lines, and by the presence or absence of perpendicular lines.

10. Which specific quadrilateral is formed by the intersection of the four lines? Explain how you know.

11. What is the slope of a line parallel to the line with equation $y = \frac{4}{5}x - 7$?

12. What is the slope of a line perpendicular to the line with equation $y = -2x + 1$?

13. Consider the lines described below.
   - line $a$: a line with the equation $y = \frac{3}{5}x - 2$
   - line $b$: a line containing the points $(-6, 1)$ and $(3, -14)$
   - line $c$: a line containing the points $(-1, 8)$ and $(4, 11)$
   - line $d$: a line with the equation $-5x + 3y = 6$

   Use the vocabulary from the lesson to write a paragraph that describes the relationships among lines $a$, $b$, $c$, and $d$.

14. What is the slope of a line parallel to the line with equation $y = -3$? What is the slope of a line perpendicular to the line with equation $y = -3$? Explain your reasoning.

15. **Construct viable arguments.** Suppose you are given the equation of a line and the coordinates of a point not on the line. Explain how to write the equation of a parallel line that passes through the point.
Lesson 8-2 Practice

16. What is the equation of a line parallel to \( y = -3x + 5 \) that passes through point \((6, 8)\)?

17. What is the equation of a line perpendicular to \( y = \frac{1}{4}x - 3 \) that passes through point \((-2, 4)\)?

Use the coordinate plane for Items 24–26.

18. What is the equation of the line parallel to line \( \ell \) that passes through point \((3, 3)\)?

19. What is the equation of the line perpendicular to line \( \ell \) that passes through point \((3, 3)\)?

20. How could you check that your answer to Item 19 is reasonable?

21. Reason abstractly and quantitatively. \( \triangle ABC \) has vertices at the points \( A(-2, 3), B(7, 4), \) and \( C(5, 22) \). What kind of triangle is \( \triangle ABC \)? Explain your answer.
ACTIVITY 8 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 8-1

1. Determine the slope of the line that contains the points with coordinates (1, 5) and (−2, 7).

2. Line \( m \) contains the points with coordinates (−4, 1) and (5, 8), and line \( n \) contains the points with coordinates (6, −2) and (10, 7). Are the lines parallel, perpendicular, or neither? Justify your answer.

A drafter finished the first stage of a drawing of a metal part. Use the drawing for Items 3–6.

![Diagram of a metal part]

3. The drafter needs to confirm that \( \overline{AG} \parallel \overline{CD} \). Are these segments parallel? Explain.

4. The drafter also needs to confirm that \( \overline{BF} \perp \overline{FD} \). Are these segments perpendicular? Explain.

5. Which segments in the diagram are perpendicular to \( \overline{GE} \)?

6. Name a segment that is neither parallel nor perpendicular to \( \overline{BC} \).

7. \( \overline{PQ} \) has a slope of \( \frac{1}{3} \). What is the slope of a line perpendicular to \( \overline{PQ} \)?
   - A. −3
   - B. \( -\frac{1}{3} \)
   - C. \( \frac{1}{3} \)
   - D. 3

8. \( \overline{VW} \) has a slope of 5. What is the slope of a line parallel to \( \overline{VW} \)?
   - A. −5
   - B. \( -\frac{1}{5} \)
   - C. \( \frac{1}{5} \)
   - D. 5

Use the diagram for Items 9 and 10.

![Diagram with labeled points]

9. What is the slope of a line parallel to line \( s \)?

10. What is the slope of a line perpendicular to line \( s \)?

Lesson 8-2

11. Which of the following is NOT an equation for a line parallel to \( y = \frac{1}{2} x - 6 \)?
   - A. \( y = \frac{2}{4} x + 6 \)
   - B. \( y = 0.5x - 3 \)
   - C. \( y = \frac{1}{2} x + 1 \)
   - D. \( y = 2x - 4 \)

12. Determine the slope of a line perpendicular to the line with equation \( 6x - 4y = 30 \).
13. Which of the following represents a line parallel to line $t$?
   - A. $y = -x + 7$
   - B. $y = -\frac{1}{3}x + 5$
   - C. $y = \frac{1}{3}x - 2$
   - D. $y = x - 4$

14. Which of the following represents a line perpendicular to line $t$?
   - A. $y = -3x + 4$
   - B. $y = -\frac{1}{3}x - 1$
   - C. $y = \frac{1}{3}x - 5$
   - D. $y = 3x + 4$

15. Consider the line with equation $y = -2x + 2$.
   a. Can you write the equation of a line through point $(3, -4)$ that is parallel to the given line? Explain.
   b. Can you write the equation of a line through point $(3, -4)$ that is perpendicular to the given line? Explain.

16. Suppose you are given the equation of a line and the coordinates of a point not on the line. Explain how to write the equation of a perpendicular line that passes through the point.

17. What is the equation of a line through point $(0, 5)$ that is parallel to the line with equation $y = x - 7$?

18. What is the equation of a line through point $(-4, 5)$ that is perpendicular to the line with equation $y = -6x + 4$?

19. Use the Distance Formula to show that $\overline{JM} \cong \overline{MK}$.

20. Is $\overline{LM}$ the perpendicular bisector of $\overline{JK}$? Explain how you know.

**MATHEMATICAL PRACTICES**
Make Sense of Problems and Persevere in Solving them

21. A drafter is drawing quadrilateral $ABCD$ as part of an architectural blueprint. She needs to position point $C$ so that $\overline{BC} \parallel \overline{AD}$ and $\overline{CD} \perp \overline{AD}$.

   a. What should be the coordinates of point $C$? Explain how you determined your answer.
   b. Describe how you can check that your answer is reasonable.
The first hill of the Steel Dragon 2000 roller coaster in Nagashima, Japan, drops riders from a height of 318 ft. A portion of this first hill has been transposed onto a coordinate plane and is shown to the right.

1. The structure of the supports for the hill consists of steel beams that run parallel and perpendicular to one another. The endpoints of the beam shown by $\overline{BC}$ are (0, 150) and (120, 0). The endpoints of the beam shown by $\overline{AD}$ are (0, 125) and (100, 0).
   a. Verify and explain why the two beams are parallel.
   b. If $\angle DAB = 125^\circ$, what is the measure of $\angle CBA$? Justify your reasoning.

2. Determine the equations of the lines containing the beams from Item 1, and explain how the equations of the lines can help you determine that the beams are parallel.

3. A third support beam is perpendicular to the beam shown by $\overline{BC}$. If marked on the coordinate plane, the line containing this beam would pass through the point (60, 75). What is the equation of the line containing this third beam? Explain how you determined your answer.

The diagram below shows a section of the steel support structure of a roller coaster. Use the diagram for Items 4–6.

4. Given $\overline{JK} \parallel \overline{PL}$, $m\angle JKL = (10x + 5)^\circ$ and $\angle PLK = (12x - 1)^\circ$, what is the measure, in degrees, of $\angle JKL$ and $\angle PLK$? Explain how you determined your answer.

5. Explain how an engineer could determine whether $\overline{PN} \parallel \overline{LM}$ by measuring two angles in the diagram.

6. Write a two-column proof.
   Given: $\overline{JK} \parallel \overline{PL}$, $\angle 1 \cong \angle 2$
   Prove: $\angle 3 \cong \angle 4$
### Scoring Guide

<table>
<thead>
<tr>
<th>Mathematics Knowledge and Thinking (Items 1, 2, 3, 4, 5)</th>
<th>Exemplary</th>
<th>Proficient</th>
<th>Emerging</th>
<th>Incomplete</th>
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<td>A functional understanding of slopes of parallel and perpendicular lines</td>
<td>Partial understanding of slopes of parallel and perpendicular lines</td>
<td>Little or no understanding of slopes of parallel and perpendicular lines</td>
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<td>Clear and accurate understanding of pairs of angles formed by parallel lines cut by a transversal</td>
<td>Adequate understanding of pairs of angles formed by parallel lines cut by a transversal</td>
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<td>An appropriate and efficient strategy that results in a correct answer</td>
<td>A strategy that may include unnecessary steps but results in a correct answer</td>
<td>A strategy that results in some incorrect answers</td>
<td>No clear strategy when solving problems</td>
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<td>Mostly correct understanding of determining slope and creating an equation to represent a line</td>
<td>Partial understanding of determining slope and creating an equation to represent a line</td>
<td>Inaccurate or incomplete understanding of determining slope and creating an equation to represent a line</td>
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<td>A functional understanding of creating an equation to represent the relationship between a pair of angles formed by parallel lines cut by a transversal</td>
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<td>Adequate explanation of the relationships between the slopes of parallel and perpendicular lines</td>
<td>Misleading or confusing characterization of the relationships between the slopes of parallel and perpendicular lines</td>
<td>Incomplete or inaccurate characterization of the relationships between the slopes of parallel and perpendicular lines</td>
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<td>Adequate use of appropriate math terms and language to justify the relationships between angles formed by parallel lines cut by a transversal</td>
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