Attend to precision. a. three translations, each of directed line segment $\mathbf{AB}$. Notice that the transformation that occurs first in the series is in the interior of the notation, and subsequent transformations are written outside of it.

1. Attend to precision. Write the notations for these compositions of transformations. Use the points $A(0, 0)$, $B(1, 1)$, and $C(0, -1)$.
   a. a clockwise rotation of 60° about the origin, followed by a translation by directed line segment $\mathbf{AB}$
      \[ T_{(0,0)} (R_{(0,0)} 60°) \]
   b. a reflection about the line $x = 1$, followed by a reflection about the line $x = 2$
      \[ R_{x=2}(R_{x=1}) \]
   c. three translations, each of directed line segment $\mathbf{AC}$
      \[ T_{(0,-1)}(T_{(0,-1)}(T_{(0,-1)}) \]

ACTIVITY 10

ACTIVITY Standards Focus

In Activity 10, students learn about compositions of transformations both on and off the coordinate plane. They identify a sequence of transformations that will carry a pre-image to an image, and vice versa. Finally, they define congruence in terms of rigid motions. Emphasize the relationship between rigid motions and congruent figures.

Lesson 10-1

PLAN

Pacing: 1 class period

**Chunking the Lesson**

#1 #2-3 #4-5 #6 #7-9 #10

Example A
Check Your Understanding
Lesson Practice

TEACH

**Bell-Ringer Activity**

Write a function to describe each transformation in the coordinate plane.

1. a translation 2 units up and 1 unit to the left \( (x, y) \rightarrow (x - 1, y + 2) \)
2. a reflection across the $x$-axis \( (x, y) \rightarrow (x, -y) \)
3. a rotation 90° clockwise about the origin \( (x, y) \rightarrow (y, -x) \)

**Developing Math Language**

This lesson refers to composition of transformations to describe two or more transformations performed in a sequence. Ask students to predict how the order of transformations in a composition will affect the final outcome. \[\text{Sometimes order matters, sometimes not.}\]

1 Close Reading, Activating Prior Knowledge, Graphic Organizer

Emphasize the importance of paying attention to detail as students write the notations for each composition. Remind them that they have seen similar notation for the composition of functions. Guide them to see similarities. For example, how does the notation tell you the order in which the transformations are performed? \[\text{The notation is nested, with the innermost transformation performed first.}\]

If students are having trouble reading and writing the notation, encourage them to create some sort of graphic organizer or “decoder” to help them keep track of the various symbols and their arrangement.

**Common Core State Standards for Activity 10**

HSG.CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

HSG.CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
The figure shows an arrow that points from a short horizontal line through (4, 2) to its tip at (4, 10).

2. Draw the image of the arrow mapped by the composition $r_x – 4(R_{90, 090})$. Label it $A$.

3. Draw the image of the arrow mapped by a composition of the same transformations in the reverse order: $R_{180, 090}(r_x – 4)$. Label it $B$.

4. What single transformation maps the arrow to image $A$? Write the name of the transformation and its symbolic representation.

   rotation, $R_{4, 65, 30°}$

5. What single transformation maps the arrow to image $B$?

   $R_{4, 93, 30°}$

As demonstrated by Items 2 and 3, the order of transformations in a composition can affect the position and orientation of the image. And as shown by Items 4 and 5, a composition can produce the same image as a single translation, reflection, or rotation.

6. For each of these compositions, predict the single transformation that produces the same image.

   a. $T_{1,2}(T_{1,0}(T_{1,0}))$  
      $T_{2,2}$

   b. $R_{0, 30°}(R_{0, 30°})$  
      $R_{0, 180°}$

   c. $r_x – 4(r_y – 0)$  
      $R_{0, 180°}$

If students are struggling with Item 6, encourage them to “simplify the problem” by thinking about the composition of the transformations in terms of a simple figure, such as a square, in the coordinate plane. Have students describe in words how the square will be affected after each transformation in each composite. For example, in Item 6a, the first transformation to be performed, $T_{1,0}$, would move the square 1 unit to the right. Then have students try to describe in words a single transformation that produces the same image.
Lesson 10-1
Compositions of Transformations

Use the figure for Items 7–9.

7. Identify a composition of transformations that could map the arrow on the left to the image of the arrow on the right.
   Sample answer: $R_{90^\circ CW}(T_{0,0})$

8. Consider the composition you identified in Item 7 but with the transformations in reverse order. Does it still map the arrow to the same image?
   Sample answer: No, the image from this combination is located entirely in the fourth quadrant.

9. Identify a composition that undoes the mapping, meaning it maps the image of the arrow on the right to the pre-image on the left.
   Sample answer: $T_{(0,6)}(R_{90^\circ W})$

You can also find compositions of transformations away from the coordinate plane.

10. Points A, B, C, and D are points on the right-pointing arrow shown here. Predict the direction of the arrow after it is mapped by these compositions.

   a. $T_{90^\circ CW}(T_{90^\circ})$ right
   b. $T_{90^\circ CW}(R_{0,0})$ up
   c. $R_{270^\circ}(T_{90^\circ})$ right

ACTIVITY 10 Continued

7–9 Work Backward, Visualization, Note Taking, Group Presentation In Items 7 through 9, students investigate the effects of different sequences of transformations on arrow shapes in the coordinate plane. Have students work in small groups, using what they know about working backward as they undo compositions. Suggest that they make their own sketches to help them visualize the arrow after each transformation. As students complete each item, encourage them to write up any additional observations they might make, with the goal of sharing these observations with the entire class.

10 Discussion Groups, Predict and Confirm, Activating Prior Knowledge, Identify a Subtask Have students work in small groups to make predictions about the direction of the arrows. This is the first time that students are working with compositions using notation for transformations away from the coordinate plane, so suggest that they review what they learned in Activity 9 about using the notation for single transformations. As they did with previous compositions, students can perform the transformations in sequence, determining the image after each one.

Differentiating Instruction
It may be beneficial for some students to approach Items 7 through 10 kinesthetically. Since each item involves a composition of transformations applied to an arrow, students can use some sort of physical motion (standing and turning, pointing a finger) to follow the sequence of motions. As students move through each motion in the sequence, ask them to explain their reasoning behind the movement and help them to correct any misconceptions.
Example A Guess and Check, Work Backward, Summarize  The solution is presented symbolically, showing how to manipulate the notation to find the inverse combination. If students are struggling with this approach, suggest that they combine guess-and-check and work-backward strategies. Start with Triangle 1, 2, or 3, and apply the composition $T_{\text{pre}}(R_{O,90^\circ})$. If the image is $\triangle A'B'C'$, then the pre-image has been found. If not, try another triangle. After students identify the correct triangle, have them review the explanation using notation again.

Universal Access

By using an agreed-upon notation to describe compositions of transformations with precision, mathematicians transcend the boundaries of language and culture. Emphasize the importance of learning to use notation to describe compositions both on and off the coordinate plane.

Try These A

Answers

a. $T_{\text{pre}}(R_{O,90^\circ})$

b. $T_{\text{pre}}$, in which $B'C'$ is a line parallel to and midway between $B'C$ and $BC$.

MATH TIP

You can think of an inverse transformation as a “reversing” or “undoing” of the transformation. It maps the image to the pre-image.

Lesson 10-1

Compositions of Transformations

Like many functions, transformations have inverses, which are transformations that map the image back to the pre-image. The inverse of transformation $T$ is designated $T^{-1}$, and it has the property $T^{-1}(T(P)) = P$ for all points $P$. The table lists the general formulas for the inverses of translations, rotations, and reflections.

<table>
<thead>
<tr>
<th>Function</th>
<th>Notation</th>
<th>Inverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translation by directed line segment $AB$</td>
<td>$T_{AB}$</td>
<td>$(T_{AB})^{-1} = T_{BA}$</td>
</tr>
<tr>
<td>Reflection about line $l$</td>
<td>$r_l$</td>
<td>$(r_l)^{-1} = r_l$</td>
</tr>
<tr>
<td>Rotation of $m$ degrees about point $O$</td>
<td>$R_{O,m^\circ}$</td>
<td>$(R_{O,m^\circ})^{-1} = R_{O,-m^\circ}$</td>
</tr>
</tbody>
</table>

Example A

Isosceles triangle $A'B'C'$ is shown in the diagram. It is the image of the combination $T_{\text{pre}}(R_{O,90^\circ})$, in which point $O$ is the center of the triangle.

Is the pre-image shown by triangle 1, 2, or 3?

Step 1: Find the inverse combination.

$(R_{O,90^\circ})^{-1} = (R_{O,-90^\circ})$, and $(T_{\text{pre}})^{-1} = T_{\text{pre}}$

So, the inverse combination is $R_{O,-90^\circ}(T_{\text{pre}})$.

Step 2: Determine the result of the translation and rotation.

When the pre-image is translated by the directed line segment $C'B$, and then rotated $90^\circ$ clockwise about its center, the result is triangle 2.

Try These A

a. Find a composition that maps triangle 1 to triangle $A'B'C'$.

b. Find a composition that maps triangle 3 to triangle $A'B'C'$.