Honors UNIT 6 Review **NAME** Key

Matrices **Period**_________

Complete each of the following:

a) In order to add to matrices together the dimensions of each matrix must be the

same.

b) In order to multiply two matrices together the number of columns of the **first** matrix must be the same as the number of rows of the **second** matrix.

c) How do you calculate the determinant of a matrix? $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

d) What are the steps to finding the inverse of a 2x2 matrix (by hand)?

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is 2x2 then

$\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Use the following information to answer a and b: A_{2x3}, B_{2x3}, C_{3x2}, D_{4x2}, E_{2x4}

a) What pairs of matrices can be multiplied? AC BC CA CB CE DA DB DE ED

b) What pairs of matrices can be added? A+B

also each matrix can be added to itself:

A+A, B+B, C+C, D+D, E+E

Use the following matrices to complete problems 1 – 4

$$M_1 = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix} \qquad M_2 = \begin{bmatrix} 2 & 3 & 4 \\ 9 & 8 & 0 \end{bmatrix} \qquad M_3 = \begin{bmatrix} -5 & 6 \\ 3 & 4 \\ 1 & 2 \end{bmatrix}$$

$$M_4 = \begin{bmatrix} -6 & -2 \\ -3 & -1 \end{bmatrix} \qquad M_5 = \begin{bmatrix} 1 & 3 \\ 5 & -2 \end{bmatrix}$$

1) Compute the determinant of each matrix. State the inverse of each matrix. If it does not exist, specify the reason.

$det\ M_1 =$ $8-9=-1$	$det\ M_2 =$ DNE not a square matrix	$det\ M_3 =$ DNE not a square matrix	$det\ M_4 =$ $6-6=0$	$det\ M_5 =$ $-2-15=-17$
$M_1^{-1} =$ $-1\begin{bmatrix} -4 & -3 \\ -3 & -2 \end{bmatrix}$ $= \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$	$M_2^{-1} =$ DNE not a square matrix	$M_3^{-1} =$ DNE not a square matrix	$M_4^{-1} =$ DNE det=0	$M_5^{-1} =$ $\frac{-1}{17}\begin{bmatrix} -2 & -3 \\ 5 & 1 \end{bmatrix}$ $\begin{bmatrix} \frac{2}{17} & \frac{3}{17} \\ -\frac{5}{17} & \frac{-1}{17} \end{bmatrix}$

2) Evaluate each of the following. If the operation cannot be completed, state the reason.

$M_4 + M_1$ $\begin{bmatrix} -6 & -2 \\ -3 & -1 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$ $= \begin{bmatrix} -8 & 1 \\ 0 & -5 \end{bmatrix}$	$M_1 + M_2$ DNE not the same dimension	$M_1 - M_5$ $\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ 5 & -2 \end{bmatrix}$ $= \begin{bmatrix} -3 & 0 \\ -2 & -2 \end{bmatrix}$	$M_1 - 2M_4$ $\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix} - \begin{bmatrix} -12 & -4 \\ -6 & -2 \end{bmatrix}$ $= \begin{bmatrix} 10 & 7 \\ 9 & -2 \end{bmatrix}$
$M_1 \cdot M_2$ $\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \\ 9 & 8 & 0 \end{bmatrix}$ $\begin{bmatrix} 23 & 18 & -8 \\ 30 & -23 & 12 \end{bmatrix}$	$M_2 \cdot M_3$ $\begin{bmatrix} 2 & 3 & 4 \\ 9 & 8 & 0 \end{bmatrix}\begin{bmatrix} -5 & 6 \\ 3 & 4 \\ 1 & 2 \end{bmatrix}$ $= \begin{bmatrix} 3 & 32 \\ -21 & 86 \end{bmatrix}$	$M_1 \cdot M_3$ 2x2 3x2 DNE number of columns in 1st matrix ≠ # of rows in 2nd matrix	$M_4 \cdot M_5$ $\begin{bmatrix} -6 & -2 \\ -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 5 & -2 \end{bmatrix}$ $= \begin{bmatrix} -16 & -14 \\ -8 & -7 \end{bmatrix}$

3) Manually find the inverse of the following matrices. SHOW ALL WORK.

a. $\begin{bmatrix} 3 & -4 \\ 2 & 6 \end{bmatrix}$ $18+8=26$ $\frac{1}{26}\begin{bmatrix} 6 & 4 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{13} & \frac{2}{13} \\ -\frac{1}{13} & \frac{3}{26} \end{bmatrix}$	b. $\begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}$ $-8+3=-5$ $-\frac{1}{5}\begin{bmatrix} 4 & -3 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} -\frac{4}{5} & \frac{3}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$

4) Find the inverse of each matrix below using a calculator:

a. $\begin{bmatrix} 4 & 3 & 1 \\ -1 & 0 & -3 \\ 3 & 1 & -3 \end{bmatrix}$ $\begin{bmatrix} -3/25 & -2/5 & 9/25 \\ 12/25 & 3/5 & -11/2 \\ 1/25 & -1/5 & -3/25 \end{bmatrix}$	b. $\begin{bmatrix} 2 & 1 & 1 \\ 2 & -1 & 2 \\ 1 & 4 & -3 \end{bmatrix}$ $\begin{bmatrix} -5/7 & 1 & 3/7 \\ 8/7 & -1 & -2/7 \\ 9/7 & -1 & -4/7 \end{bmatrix}$

5) RETAIL: Current Fashions buys shirts, jeans, and shoes from a manufacturer, marks them up, and then sells them. The table shows the purchase price and the selling price.

Item	Purchase Price	Selling Price
Shirts	\$15	\$35
Jeans	\$25	\$55
Shoes	\$30	\$85

A) Write a matrix for the information: $\begin{array}{cc} & \begin{array}{ccc} S & J & Sh \end{array} \\ \begin{array}{c} PP \\ SP \end{array} & \begin{bmatrix} 15 & 25 & 30 \\ 35 & 55 & 85 \end{bmatrix} \end{array}$	B) Show a matrix that would increase the purchase price by 5% by and the selling price by 7%. $\begin{bmatrix} .05 & 0 \\ 0 & .07 \end{bmatrix}$

$$\begin{bmatrix} .05 & 0 \\ 0 & .07 \end{bmatrix}\begin{bmatrix} 15 & 25 & 30 \\ 35 & 55 & 85 \end{bmatrix}$$

6) Tyrone Davidson runs the local Ty-D-Bowl Family Bowling Center. He needs to buy new bowling balls for his center, according to the following cost schedule: 8 pound balls at \$35 each; 12 pound balls at \$40 each; 16 pound balls at \$55 each. Shipping costs \$0.50 per pound. If Tyrone orders 42 new bowling balls costing \$1950 plus \$276 for shipping, find the number of each type of ball he ordered.

x = # of 8 lb balls
y = # of 12 lb balls
z = # of 16 lb balls

Matrix Equation:

$x + y + z = 42$
$35x + 40y + 55z = 1950$
$4x + 6y + 8z = 276$

$$\begin{bmatrix} 1 & 1 & 1 \\ 35 & 40 & 55 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 42 \\ 1950 \\ 276 \end{bmatrix}$$

Solution:

9 8 lb balls
12 12 lb balls
21 16 lb balls

7) Create the inverse matrix and solve the matrix equation. Be able to identify the coefficient matrix (A), variable matrix (X), and constant matrix (B). If there is not a unique solution, state whether there are infinitely many solutions (parameterize the solution) or no solution.

a) $\begin{cases} 3x - 2y = -6 \\ x + y = -2 \end{cases}$

Matrix Equation: $\begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ -2 \end{bmatrix}$

Inverse matrix: $\begin{bmatrix} 1/5 & 2/5 \\ -1/5 & 3/5 \end{bmatrix}$

Solution: $(-2, 0)$

b) $\begin{cases} 1.5x + 2y = 20 \\ 25 = -2.5x + 5y \end{cases}$

Matrix Equation: $\begin{bmatrix} 1.5 & 2 \\ 2.5 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ -25 \end{bmatrix}$

Inverse matrix: $\begin{bmatrix} 2/5 & 4/25 \\ 1/5 & -3/25 \end{bmatrix}$

Solution: $(4, 7)$

f) $\begin{cases} x - 2y + z = 7 \\ 3x + y - z = 2 \\ 2x + 3y + 2z = 7 \end{cases}$

Matrix Equation: $\begin{bmatrix} 1 & -2 & 1 \\ 3 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \\ 7 \end{bmatrix}$

Solution: $(2, -1, 3)$

g) $\begin{cases} x + 2y + 3z = 8 \\ 4x - 7z = 3 \\ z - 2x = 5y + 5 \end{cases}$

Matrix Equation: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & -7 \\ -2 & -5 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ 5 \end{bmatrix}$

Solution: $(401/75, -196/75, 194/75)$

h) $\begin{cases} 5x - 2y + z = 3 \\ 4x - 4y - 8z = 2 \\ -x + y + 2z = -3 \end{cases}$

Matrix Equation: $\begin{bmatrix} 5 & -2 & 1 \\ 4 & -4 & -8 \\ -1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}$

Solution: no solution

i) $\begin{cases} x + y + z = 2 \\ 2x + 2y + 2z = 4 \\ -3x - 3y = 3z - 6 \end{cases}$

Matrix Equation: $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -3 & -3 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}$

Solution: $x + y + z = 2$ $\quad (x, x - z + 2, z)$

$y = x - z + 2$

8) Given the coordinate matrix of a triangle

$$\begin{bmatrix} -4 & 1 & -2 \\ -4 & -3 & -5 \end{bmatrix} + \begin{bmatrix} -3 & -3 & -3 \\ -4 & -4 & -4 \end{bmatrix}$$

Find the coordinate matrix after the triangle is translated 3 units to the left and 4 units down.

$$\begin{bmatrix} -7 & -2 & -5 \\ -8 & -7 & -9 \end{bmatrix}$$

9) a) The ordered pair (-3, -6) is rotated 120° clockwise about the origin. What is the ordered pair representing the rotation?

$$\begin{bmatrix} \cos 120 & \sin 120 \\ -\sin 120 & \cos 120 \end{bmatrix}\begin{bmatrix} -3 \\ -6 \end{bmatrix} = \begin{bmatrix} -1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{bmatrix}\begin{bmatrix} -3 \\ -6 \end{bmatrix} =$$

exact $\begin{bmatrix} 3/2 - 3\sqrt{3} \\ \frac{3\sqrt{3}+3}{2} \end{bmatrix}$

b) A point that has been rotated 45° counterclockwise about the origin is represented by the ordered pair (3, 2). What matrix would be used to return this point to its original position? What is the ordered pair of the point prior to rotation?

approx $\begin{bmatrix} -3.696 \\ 3.598 \end{bmatrix}$

10) A polygon has one vertex at (6, -9). It is transformed with a horizontal stretch by a factor of 2 and a vertical shrink of one third. What are the coordinates of the new vertex?

$$\begin{bmatrix} 2 & 0 \\ 0 & 1/3 \end{bmatrix}\begin{bmatrix} 6 \\ -9 \end{bmatrix} = \begin{bmatrix} 12 \\ -3 \end{bmatrix}$$

(12, -3)

11) Given a triangle with the following vertices, find the area using the determinant formula.

a) (-1, 6), (4, 7), and (8, -6)

(4,7), (-1,6), (8,-6)

$$\frac{1}{2}\left(\begin{vmatrix} 4 & -1 \\ 7 & 6 \end{vmatrix} + \begin{vmatrix} -1 & 8 \\ 6 & -6 \end{vmatrix} + \begin{vmatrix} 8 & 4 \\ -6 & 7 \end{vmatrix}\right)$$

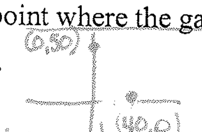
$\frac{1}{2}(31 + -42 + 80)$

$1/2(69) = 34.5$ sq units

b) (-1, -3), (4, 2), and (7, 3)

(7,3), (4,2), (-1,-3)

12) A gardener is trying to find a triangular area behind his house that encloses 1750 square feet. He has placed the first two fence posts at (0, 50) and at (40, 0). The final fence post is on the property line at y = 100. Find the point where the gardener can place the final fence post.



(30,100)

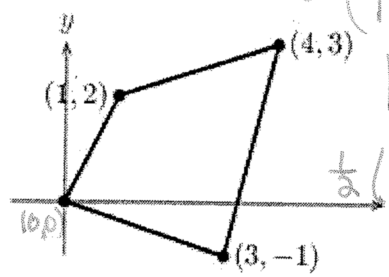
$$1/2\left(\begin{vmatrix} x & 0 \\ 100 & 50 \end{vmatrix} + \begin{vmatrix} 0 & 40 \\ 50 & 0 \end{vmatrix} + \begin{vmatrix} 40 & x \\ 0 & 100 \end{vmatrix}\right) = 1750$$

$1/2(50x + 2000) = 1750$

$25x = 750$

$x = 30$

13) Find the area of the quadrilateral shown below.



$$\frac{1}{2}\left(\begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 3 \\ 0 & -1 \end{vmatrix} + \begin{vmatrix} 3 & 4 \\ -1 & 3 \end{vmatrix}\right)$$

$\frac{1}{2}(5 + 13) = \frac{1}{2}(18)$

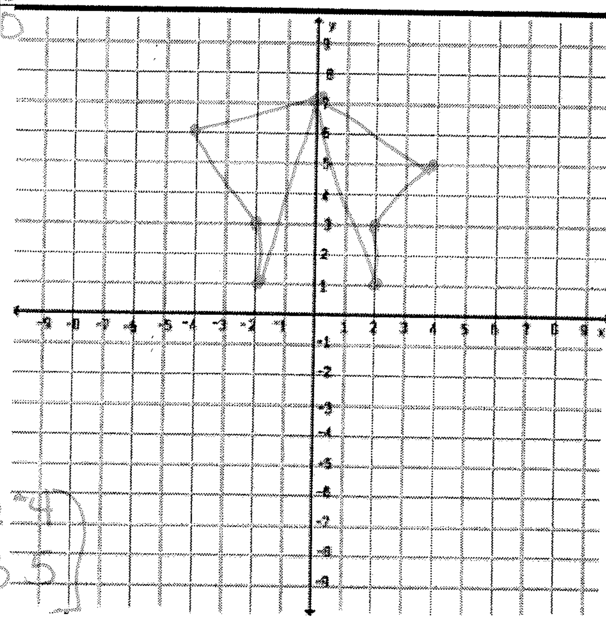
= 9 sq units

14) Let matrix A below represent a set of ordered pairs, where the first row represents the x-values and the second row of the matrix represents the y-values. On graph paper, sketch the polygon formed by connecting each of the coordinates listed in the matrix. (Make sure to close the polygon)

$$A = \begin{bmatrix} 2 & 0 & 2 & 4 \\ 1 & 7 & 3 & 5 \end{bmatrix}$$

Let matrix B be the matrix below. Find BA and graph the resulting set of ordered pairs the matrix represents.

$$B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 & 2 & 4 \\ 1 & 7 & 3 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 0 & -2 & -4 \\ 1 & 7 & 3 & 5 \end{bmatrix}$$

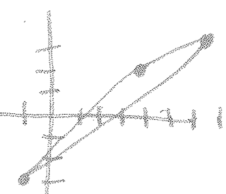


11b

$$\frac{1}{2}\left(\begin{vmatrix} 7 & 4 \\ 3 & 2 \end{vmatrix} + \begin{vmatrix} 4 & -1 \\ 2 & -3 \end{vmatrix} + \begin{vmatrix} -1 & 7 \\ -3 & 3 \end{vmatrix}\right)$$

$\frac{1}{2}(2 + -10 + 18)$

$1/2(10) = 5$ sq units



9b

to set back to original rotate 45° clockwise

$$\begin{bmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 5\sqrt{2}/2 \\ -\sqrt{2}/2 \end{bmatrix}$$

original pt