**UNIT 6 Review - Honors Precalc** NAME Key
**Matrices** Period 3

Use the following matrices to complete problems 1 - 4

$M_1 = \begin{bmatrix} -1 & 2 \\ 2 & -5 \end{bmatrix}$ 2x2 $M_2 = \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 0 \end{bmatrix}$ 2x3 $M_3 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ 3x2 $M_4 = \begin{bmatrix} -5 & -2 \\ -2 & -1 \end{bmatrix}$ $M_5 = \begin{bmatrix} 6 & 2 \\ 4 & -1 \end{bmatrix}$

2 $M_4 = \begin{bmatrix} -10 & -4 \\ -4 & -2 \end{bmatrix}$

1) Evaluate each of the following. If the operation cannot be completed, write DNE (does not exist) and state the reason.

$M_1 + M_1$ $\begin{bmatrix} -2 & 4 \\ 4 & -10 \end{bmatrix}$

$M_1 + M_2$ DNE dimension mismatch

$M_1 + M_3$ DNE dimension mismatch

$M_1 - M_4$ $\begin{bmatrix} 4 & 4 \\ 4 & -4 \end{bmatrix}$

$M_1 + 2M_4$ $\begin{bmatrix} -11 & -2 \\ -2 & -7 \end{bmatrix}$

$M_1 \times M_2$ $\begin{bmatrix} 11 & 12 & -7 \\ -30 & -33 & 14 \end{bmatrix}$

~~$M_1 \times M_2$~~

$M_2 \times M_1$ DNE # of col's in $M_2$ ≠ # of rows in $M_1$

$M_2 \times M_3 = \begin{bmatrix} 58 & 76 \\ 35 & 52 \end{bmatrix}$

2) Use matrix $M_5$ and $M_4$ to show that matrix multiplication is not commutative.

$M_5 \cdot M_4 = \begin{bmatrix} 6 & 2 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} -5 & -2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} -34 & -14 \\ -18 & -7 \end{bmatrix}$

$M_4 \cdot M_5 = \begin{bmatrix} -5 & -2 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} 6 & 2 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} -38 & -8 \\ -16 & -3 \end{bmatrix}$

$\begin{bmatrix} -34 & -14 \\ -18 & -7 \end{bmatrix} \neq \begin{bmatrix} -38 & -8 \\ -16 & -3 \end{bmatrix}$

$M_5 M_4 \neq M_4 M_5$ Mult is not comm

3) Show that $M_1$ and $M_4$ are inverses WITHOUT finding the inverse of each.

$\begin{bmatrix} -1 & 2 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} -5 & -2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

and

$\begin{bmatrix} -5 & -2 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 2 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Since $M_1 M_4 = I$ and $M_4 M_1 = I$

$M_1$ and $M_4$ are inverses

4) a) State the determinant of each matrix. If it does not exist, state the reason.

$\det(M_1) = 1$ $\det(M_2)$ DNE not square $\det(M_3)$ DNE

$\det(M_4) = 1$ $\det M_5 = -14$

b) Find the inverse of any matrix for which it is possible to have an inverse. $M_2$ + $M_3$ have no inverse det DNE not square

$M_1^{-1} = \begin{bmatrix} -5 & -2 \\ -2 & -1 \end{bmatrix}$ $M_5^{-1} = \frac{1}{14}\begin{bmatrix} -1 & -2 \\ -4 & 6 \end{bmatrix}$

$M_4^{-1} = \begin{bmatrix} -1 & 2 \\ 2 & -5 \end{bmatrix} = \begin{bmatrix} 1/14 & 1/7 \\ 2/7 & -3/7 \end{bmatrix}$

5) Write the multiplicative identity for a 4 x 4 matrix.

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

6) Write a singular matrix (a matrix that does not have an inverse) for a 2 x 2 matrix that does not contain any zeros.

Answers may vary. The determinant of your matrix should be zero.

Example

$A = \begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$

$\det(A) = 12 - 12 = 0$

A has no inverse

7) Find 2 square matrices A and B, where neither are the identity matrix, but AB DOES equal BA.

Answers vary. They should be inverses

Ex.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$ $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

8) Manually find the inverse of the following matrix:

$A = \begin{bmatrix} 2 & -4 \\ 3 & 7 \end{bmatrix}$

$\det A = 14 - -12 = 26$

$A^{-1} = \frac{1}{26}\begin{bmatrix} 7 & 4 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 7/26 & 2/13 \\ -3/26 & 1/13 \end{bmatrix}$

9) RETAIL: Current Fashions buys shirts, jeans, and shoes from a manufacturer, marks them up, and then sells them. The table shows the purchase price and the selling price.

A) Write a matrix for the purchase price.

B) Write a matrix for the selling price

C) Use matrix operations to find the profit on 1 shirt, 1 pair of jeans, and 1 pair of shoes.

D) Show a matrix that would increase both the purchase and selling price increase by 5%.

A) PP $A = \begin{matrix} S \\ J \\ Sh \end{matrix}\begin{bmatrix} 15 \\ 25 \\ 30 \end{bmatrix}$ B) SP $B = \begin{matrix} S \\ J \\ Sh \end{matrix}\begin{bmatrix} 35 \\ 55 \\ 85 \end{bmatrix}$ C) $B - A = P$ P $P = \begin{matrix} S \\ J \\ Sh \end{matrix}\begin{bmatrix} 20 \\ 30 \\ 55 \end{bmatrix}$

| Item | Purchase Price | Selling Price |
|---|---|---|
| Shirts | $15 | $35 |
| Jeans | $25 | $55 |
| Shoes | $30 | $85 |

D) $\begin{bmatrix} 1.05 & 0 & 0 \\ 0 & 1.05 & 0 \\ 0 & 0 & 1.05 \end{bmatrix}$ 3x3 $\begin{bmatrix} 15 & 35 \\ 25 & 55 \\ 30 & 85 \end{bmatrix}$ 3x2

the increased prices

$\begin{bmatrix} 15.75 & 36.75 \\ 26.25 & 57.57 \\ 31.50 & 89.25 \end{bmatrix}$

**10) A cinema has three theatre rooms. Each theatre room shows a different movie. The number of people attending each movie is shown.**

| Type of Movie | Theatre 1 | Theatre 2 | Theatre 3 |
|---|---|---|---|
| Matinee | 42 | 35 | 48 |
| Evening | 31 | 43 | 73 |

a) Write a 2 x 3 matrix for the number of customers in each theatre.

T1 T2 T3

Answer: C = $\begin{matrix} m \\ E \end{matrix}\begin{bmatrix} 42 & 35 & 48 \\ 31 & 43 & 73 \end{bmatrix}$

b) Add the numbers in row 2 and interpret the results. $31 + 43 + 73$ $3 + 1 = 74$ 74

Sum: 147

Interpretation:

The number of people in all 3 theatres for the evening performance

11) **Alex is in charge of stocking sweatshirts for the concession stand at the high school football game. The number of sweatshirts needed for the regular season game is listed in the matrix. Alex plans to triple the number of sweatshirts stocked for the playoff game.**

| Size | small | medium | large |
|---|---|---|---|
| Child | 10 | 10 | 15 |
| Adult | 25 | 35 | 45 |

a) Write a matrix A to represent the regular season stock.

S M L

$A = \begin{matrix} C \\ A \end{matrix}\begin{bmatrix} 10 & 10 & 15 \\ 25 & 35 & 45 \end{bmatrix}$ $45 \div 3 = 15$

b) What scalar can be used to determine a matrix M to represent the new numbers? Find M. $3A = M$

3

c) What is M – A? (find it). What does it represent in this situation?

$\begin{bmatrix} 30 & 30 & 45 \\ 75 & 105 & 135 \end{bmatrix} - \begin{bmatrix} 10 & 10 & 15 \\ 25 & 35 & 45 \end{bmatrix} = \begin{bmatrix} 20 & 20 & 30 \\ 50 & 70 & 90 \end{bmatrix}$

12) Solve the following system of equations by setting up a **matrix equation.** Identify the coefficient matrix, the variable matrix, and the constant matrix. Then solve the matrix equation (you may use your calculator)

$x+2y+3z=8$

$4x-7z=3$

$z-2x=5y+5$

$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & -7 \\ -2 & -5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ 5 \end{bmatrix}$ Matrix Equation

coefficient matrix | variable matrix | constant matrix

$\left(\frac{401}{75}, \frac{-196}{75}, \frac{197}{75}\right)$ Solution

13) Solve each of the following. If there is not a unique solution, state whether there are infinitely many solutions or no solution.

a) $x-y+z=10$
$-3x+3y-z=13$
$2x+5y+8z=42$
$\left(-\frac{375}{14}, \frac{-107}{7}, \frac{43}{2}\right)$

b)
$x+y+z=15$
$2x-3y+4z=13$ no solution
$3x-2y+5z=18$

c)
$2x-y-z=3$
$x+4y+9z=5$

$x + 5/9 z = 14/9$
$y + 19/9 z = 7/9$

TMS

$\left(-\frac{5}{9}z+\frac{17}{9}, \frac{-19}{9}z+\frac{7}{9}, z\right)$

14) Find the equation of a parabola (with a vertical axis of symmetry) that passes through the three points (-2, 4), (1, 1), (5, 17). $y = ax^2+bx+c$

$4 = 4a-2b+c$

$1 = a+b+c$

$17 = 25a+5b+c$

$y = \frac{5}{7}x^2 - \frac{2}{7}x + \frac{4}{7}$

15) Tyrone Davidson runs the local Ty-D-Bowl Family Bowling Center. He needs to buy new bowling balls for his center, according to the following cost schedule:

$x$ = # of 8 pound balls: \$35 each — shipping cost $8(.5) = 4$
$y$ = # of 12 pound balls: \$40 each $12(.5) = 6$
$z$ = # of 16 pound balls: \$55 each $(16)(.5) = 8$

Shipping costs \$0.50 per pound.

If Tyrone orders 42 new bowling balls costing \$1950 plus \$276 for shipping, find the number of each type of ball he ordered.

$\begin{cases} x+y+z=42 \\ 35x+40y+55z=1950 \\ 4x+6y+8z=276 \end{cases}$

9 8lb balls
12 12lb balls
21 16lb balls

16) Find the value of x and y.

$\begin{bmatrix} 3x & y+6 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} -12 & -3 \\ y+9 & 5 \end{bmatrix}$

$3x = -12$ $\quad y+6 = -3$

$x = -4$ $\quad y = -9$

17) Find the value of x and y.

$\begin{bmatrix} x^2-2 & y+7 \\ y^2 & x+4 \end{bmatrix} = \begin{bmatrix} 2 & 10 \\ 9 & 2 \end{bmatrix}$

$x^2-2 = 2$ $\quad y+7 = 10$

$x^2 = 4$ $\quad y = 3$

$x = \pm 2$

and

$x+4 = 2$

$x = -2$

only

$x = -2$ and $y = 3$

$x = -2$