I. Order of matrices

A. 2×2
B. 2×2
C. 2×2
D. 2×2
E. 2×2
F. 2×2
G. 2×2
H. 3×3
I. 3×3
J. 2×3

II. 1. $A=\begin{bmatrix}6 & 2\\ -1 & 5\end{bmatrix}$ $\det A = 30 - -2 = 32$

2. $B=\begin{bmatrix}4 & 0\\ 7 & 5\end{bmatrix}$ $|B| = 20-0 = 20$

3. $H=\begin{bmatrix}0 & 1 & 2\\ -1 & 3 & 2\\ 2 & 8 & 3\end{bmatrix}$ $|H| = -21$

4. det J: not possible
J is not square.

III 1. $A^{-1} = \frac{1}{32}\begin{bmatrix}5 & -2\\ 1 & 6\end{bmatrix} = \begin{bmatrix}5/32 & -1/16\\ 1/32 & 3/16\end{bmatrix}$

2. $B^{-1} = \frac{1}{20}\begin{bmatrix}5 & 0\\ -7 & 4\end{bmatrix} = \begin{bmatrix}1/4 & 0\\ -7/20 & 1/5\end{bmatrix}$

3. $H^{-1} = \begin{bmatrix}1/3 & -13/21 & 4/21\\ -1/3 & 4/21 & 2/21\\ 2/3 & -2/21 & -1/21\end{bmatrix}$

4. not possible

IV 1. $A^{-1} = \frac{1}{-23}\begin{bmatrix}-5 & -4\\ -2 & 3\end{bmatrix} = \begin{bmatrix}5/23 & 4/23\\ 2/23 & -3/23\end{bmatrix}$

2. no inverse, $\det(B)=0$

3. $C^{-1} = \frac{1}{2}\cdot\begin{bmatrix}-1 & 1\\ -1 & -1\end{bmatrix} = \begin{bmatrix}-1/2 & 1/2\\ -1/2 & -1/2\end{bmatrix}$

4. no inverse; D is not square

V. 1. $\begin{bmatrix}6 & 2\\ -1 & 5\end{bmatrix}\begin{bmatrix}-5.7 & 1.6\\ 3.2 & 2\end{bmatrix} = \begin{bmatrix}-27.8 & 13.6\\ 21.7 & 8.4\end{bmatrix}$

2. $HI = \begin{bmatrix}8 & -47 & 7\\ -6 & -41 & -6\\ 19 & -51 & 26\end{bmatrix}$

3. CI not possible
2×(2 3)×3
no match

4. not possible
not the same dimensions

5. $A+D = \begin{bmatrix}9 & 2\\ 0 & 3\end{bmatrix}$

6. $E-F = \begin{bmatrix}13.75 & 2.25\\ -12 & -7.5\end{bmatrix}$

7. $-2C = \begin{bmatrix}-6 & 2\\ -22 & 6\end{bmatrix}$

8. $2A+3D = \begin{bmatrix}21 & 4\\ 1 & 4\end{bmatrix}$

9. $3C-2A = \begin{bmatrix}-3 & -7\\ 35 & -19\end{bmatrix}$

VI 1. $4x=-24$ $x=-6$ $\quad y+2=15$ $y=13$

2. $-5x=20$ $x=-4$ $\quad 7y=21$ $y=3$

3. $2x=16$ $x=8$ $\quad -y=-3$ $y=3$

4. $-3y=-3$ $y=1$ $\quad .5x=5$ $x=5/.5$ $x=10$

5. $5x+y=13$, $2x+y=7$
$2(2)+y=7$
$y=3$
$5x+y=13$, $-2x-y=-7$
$3x=6$
$x=2$

6. $3x+5+5y=-10$
$x-1+3y+1=-9$

$3x+5y=-15$, $x+3y=-8$
$x-\frac{27}{4}=\frac{-32}{4}$
$x=-5/4$

$3x+5y=-15$, $-3x-9y=24$
$-4y=9$
$y=-9/4$

VII 1. $\begin{bmatrix} 2 & -1 & -4 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 8 \\ 1 \\ 1 \end{bmatrix}$

coefficient matrix · variable matrix · constant matrix

2. $\begin{bmatrix} -4 & 0 & 2 \\ -1 & 1 & -1 \\ -2 & 0 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 14 \\ 12 \\ 22 \end{bmatrix}$

coefficient matrix · variable matrix · constant matrix

3. $\begin{bmatrix} 5 & 3 & 2 \\ 2 & -1 & 1 \\ -2 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

coefficient matrix · variable matrix · constant matrix

4. $\begin{bmatrix} -1 & 1 & 2 \\ 2 & 3 & 1 \\ -3 & -4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 7 \\ 1 \\ 4 \end{bmatrix}$

coefficient matrix · variable matrix · constant matrix

VIII 1 $A=\left[\begin{array}{cc|c} 1 & -1 & 5 \\ 2 & -1 & 6 \end{array}\right]$

$\text{rref}(A)=\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -4 \end{array}\right]$

$(1,-4)$

2. $A=\left[\begin{array}{cc|c} 4 & 7 & 10 \\ 3 & 5 & 9 \end{array}\right]$

$\text{rref}(A)=\left[\begin{array}{cc|c} 1 & 0 & 13 \\ 0 & 1 & -6 \end{array}\right]$

$(13,-6)$

9. $A=\left[\begin{array}{ccc|c} 2 & 2 & 1 & 6 \\ 3 & 2 & 2 & 8.48 \\ 4 & 3 & 2 & 10.46 \end{array}\right]$

$\text{rref}A=\left[\begin{array}{ccc|c} 1 & 0 & 0 & .44 \\ 0 & 1 & 0 & 1.54 \\ 0 & 0 & 1 & 2.04 \end{array}\right]$

$(.44, 1.54, 2.04)$

10. $A=\left[\begin{array}{ccc|c} 2 & 1 & -3 & 2365 \\ 1 & 3 & 2 & 2045 \\ 3 & 2 & 1 & 2460 \end{array}\right]$

$(432.5, 712.5, -262.5)$

3) $A=\left[\begin{array}{cc|c} 3 & -1 & 5 \\ -2 & 1 & -4 \end{array}\right]$

$\text{rref}(A)=\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -2 \end{array}\right]$

$(1, -2)$

4) $A=\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 1 & -2 & -8 \\ -1 & 0 & 4 & 10 \end{array}\right]$

$\text{rref}A=\left[\begin{array}{ccc|c} 1 & 0 & 0 & -32 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & -8 \end{array}\right]$

$(-32, 40, -8)$

5) $A=\left[\begin{array}{ccc|c} 3 & 3 & -1 & 19 \\ 5 & 4 & -2 & 28 \\ 2 & 2 & -1 & 12 \end{array}\right]$

$\text{rref}A=\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$(4, 3, 2)$

6) $A=\left[\begin{array}{ccc|c} 1 & 1 & -1 & 10 \\ 2 & 0 & 1 & 12 \\ 0 & -1 & 1 & 3 \end{array}\right]$

$\text{rref}A=\left[\begin{array}{ccc|c} 1 & 0 & 0 & 13 \\ 0 & 1 & 0 & -17 \\ 0 & 0 & 1 & -14 \end{array}\right]$

$(13, -17, -14)$

11) $A=\left[\begin{array}{cccc|c} 1 & -5 & 2 & -1 & -18 \\ 3 & 1 & -3 & 2 & 14 \\ 4 & -2 & 1 & -1 & -1 \\ -2 & 3 & -1 & 4 & -11 \end{array}\right]$

$\text{rref}A=\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 84/85 \\ 0 & 1 & 0 & 0 & 282/85 \\ 0 & 0 & 1 & 0 & -347/85 \\ 0 & 0 & 0 & 1 & -98/17 \end{array}\right]$

$(84/85, 282/85, -347/85, -98/17)$

12. $A=\left[\begin{array}{cccc|c} 2 & -1 & 5 & 1 & -3 \\ 3 & 2 & 2 & -6 & -32 \\ 1 & 3 & 3 & -1 & -47 \\ 5 & -2 & -3 & 3 & 49 \end{array}\right]$

$\text{rref}A=\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -12 \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$

$(2, -12, -4, 1)$

13. $A=\left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 2 & -5 & -2 & 0 \\ 4 & -11 & 2 & 0 \end{array}\right]$

$\text{rref}A=\left[\begin{array}{ccc|c} 1 & 0 & -16 & 0 \\ 0 & 1 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

IMS

$x-16z=0 \quad y-6z=0$

$x=16z \quad y=6z$

$(16z, 6z, z)$

7. $A = \left[\begin{array}{ccc|c} 1 & 3 & 4 & 11 \\ 2 & 3 & 2 & 7 \\ 4 & 9 & 10 & 20 \\ 3 & -2 & 1 & 1 \end{array}\right]$

$rrefA \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

no solution

8. $A = \left[\begin{array}{ccc|c} 2 & 5 & 2 & -1 \\ 1 & 2 & -3 & 5 \\ 5 & 12 & 1 & 10 \end{array}\right]$

$rrefA = \left[\begin{array}{ccc|c} 1 & 0 & -19 & 0 \\ 0 & 1 & 8 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

no solution

14. $A = \left[\begin{array}{cccc|c} 1 & -1 & 2 & -3 & 9 \\ 4 & 0 & 11 & -10 & 46 \\ 3 & -1 & 8 & -6 & 27 \end{array}\right]$

$rrefA = \left[\begin{array}{cccc|c} 1 & 0 & 0 & -13.5 & 39 \\ 0 & 1 & 0 & -2.5 & 10 \\ 0 & 0 & 1 & 4 & -10 \end{array}\right]$

$t - 13.5w = 39$

$u - 2.5w = 10$

$v + 4w = -10$

$t = 13.5w + 39$

$u = 10 + 2.5w$

$v = -10 - 4w$

$(13.5w + 39, 10 + 2.5w, -10 - 4w, w)$