Honors PreCalculus

Name Key

Conic Section Study Guide

Identify each equation as a parabola, circle, ellipse, or hyperbola. Explain your answer.

1. circle
$(x+2)^2+(y+3)^2=18$
standard form

2. ellipse
$x^2+2y^2+2x+8y=15$
$A \neq B$
same sign

3. Parabola
$x^2-4x+4=12y-12$
only 1 variable has a squared term

4. hyperbola
$x^2-4y^2+10x-16y=-5$
$A \neq B$
different signs

5. circle
$x^2+y^2-8x+2y+13=0$
$A=B$

6. ellipse
$49x^2+16y^2=784$
$A \neq B$
same sign

Determine the standard form of the equation of the conic given the necessary components.

$(y+6)^2=-16(x-4)$ $(y-3)^2=-14(x+8.5)$

7. Parabola:
Vertex: $(4,-6)$
Focus: $(0,-6)$
opens left $p=-4$

8. Parabola:
Focus: $(-12,3)$
Directrix: $x=-5$
V $(-8.5,3)$ opens left
$p=-3.5$

9. Circle: $(x-4)^2+(y-2)^2=61$
Center $(4,2)$
Point on the circle $(-1,8)$
$r=\sqrt{(4-1)^2+(2-8)^2}=\sqrt{25+36}=\sqrt{61}$

10. Ellipse:
Center: $(7,-10)$
Vertex: $(-6,-10)$
Co-vertex: $(7,-17)$
$\frac{(x-7)^2}{169}+\frac{(y+10)^2}{49}=1$

11. Ellipse: center $(2,3)$
Vertices: $(-2,13)$ *and* $(-2,-7)$
Co-vertices: $(6,3)$ *and* $(-10,3)$
$\frac{(x+2)^2}{64}+\frac{(y-3)^2}{100}=1$

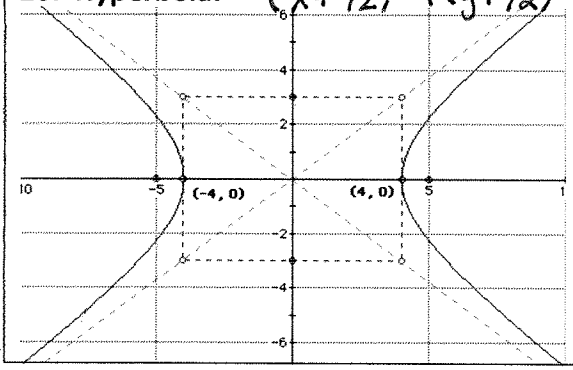
12. Hyperbola:
Foci: $(6,0)$ *and* $(-6,0)$
Vertices: $(4,0)$ *and* $(-4,0)$
$\frac{x^2}{16}-\frac{y^2}{20}=1$
$36=16$

13. Circle: C $\left(\frac{-4+3}{2}, \frac{-3+2}{2}\right)$ C$(-1/2, -1/2)$
Endpoints on diameter:
(-4, -3) and (3, 2)
$d=\sqrt{49+25}=\sqrt{74}$ $r=\sqrt{74}/2$ $r^2=74/4=37/2$

14. Hyperbola: (below)
Horizontal transverse axis
Center (-3, 2)
Slope of an asymtotes: $\pm\frac{4}{3}$

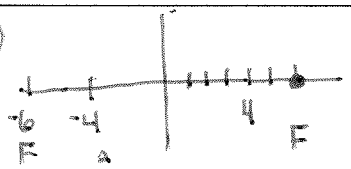
15. Ellipse (below)
Vertices: (-6, 4) and (4, 4)
Foci: (-4, 4) and (2, 4)

16. Hyperbola: $(x+1/2)^2+(y+1/2)^2=37/2$

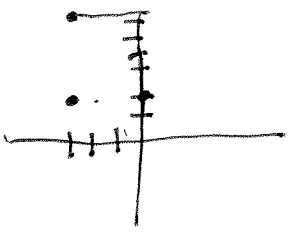


$\frac{x^2}{16}-\frac{y^2}{9}=1$

(12)



$c=16$
$a^2=36$
$c^2=a^2+b^2$
$36=16+b^2$



14. $\frac{(x+3)^2}{9}-\frac{(y-2)^2}{16}=1$

15. $\frac{(x+1)^2}{25}+\frac{(y-4)^2}{16}=1$

V F C F V C(-1,4)

$c=3$
$9=25-b^2$ $b^2=16$
$-16=-b^2$

Graph each of the following:

17.

Vertex: (-3, 5)

Focus: (-3, 6 3/4)

Directrix: y = 3 1/4

Axis of Symmetry: x = -3

5 - 1 3/4 3 1/4

$2x^2 + 12x - 14y + 88 = 0$

$2(x^2 + 6x + 9) = 14y - 88 + 18$

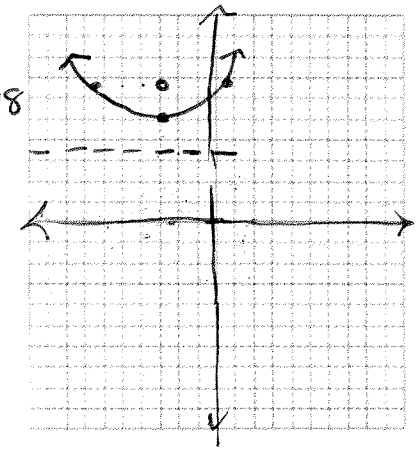
$2(x+3)^2 = 14y - 70$

$(x+3)^2 = 7y - 35$

$(x+3)^2 = 7(y-5)$

$4p = 7$ opens up

$p = 7/4$



18

Vertex: (-3, 2)

Focus: (-3, 4)

Directrix: y = 0

Axis of Symmetry: x = -3

$y^2 - 8x - 4y - 20 = 0$

$y^2 - 4y + 4 = 8x + 20 + 4$

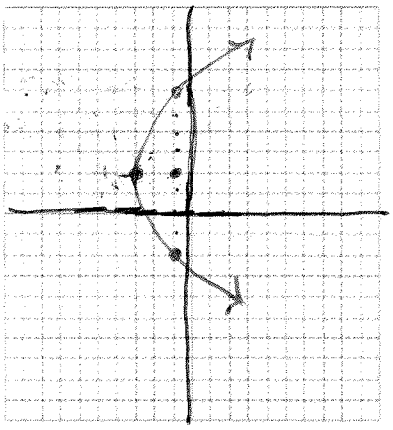
$(y-2)^2 = 8x + 24$

$(y-2)^2 = 8(x+3)$

$4p = 8$

$p = 2$

opens right

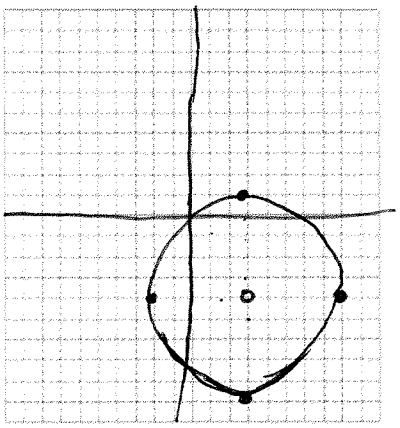


19.

Center: (3, -4)

Radius: 5

$(x-3)^2 + (y+4)^2 = 25$



20.

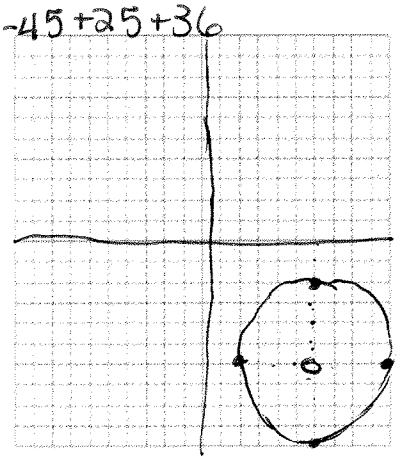
Center: (5, -6)

Radius: 4

$x^2 + y^2 - 10x + 12y = -45$

$(x^2 - 10x + 25) + (y^2 + 12y + 36) = -45 + 25 + 36$

$(x-5)^2 + (y+6)^2 = 16$



21.

$$\frac{(x+2)^2}{36} + \frac{(y+3)^2}{100} = 1$$

Center: (-2, -3)

Vertices: (-2, 7) and (-2, -13)

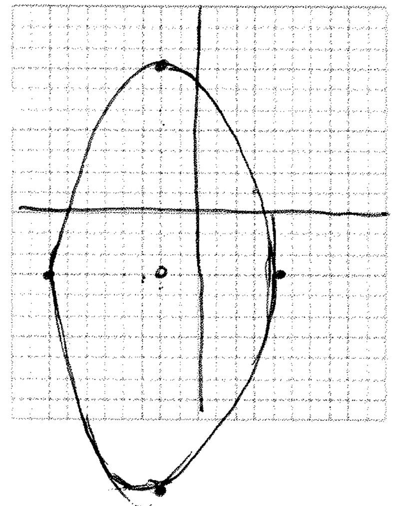
Co-Vertices: (-8, -3), and (4, -3)

Foci: (-2, 5), and (-2, -11)

Length of Major Axis: 20

Length of Minor Axis: 12

100 − 36 = 64   c = √64 = 8



48 169 36 1014 507 6084

6084 625 6709 −2484 4225

22.

$$25x^2 + 169y^2 + 250x - 2028y = -2484$$

$25(x^2+10x+25)+169(y^2-12y+36) = -2484+625+6084$

$\frac{25(x+5)^2}{4225} + \frac{169(y-6)^2}{4225} = 1$

$\frac{(x+5)^2}{169} + \frac{(y-6)^2}{25} = 1$

Center: (-5, 6)

Vertices: (-18, 6), and (8, 6)

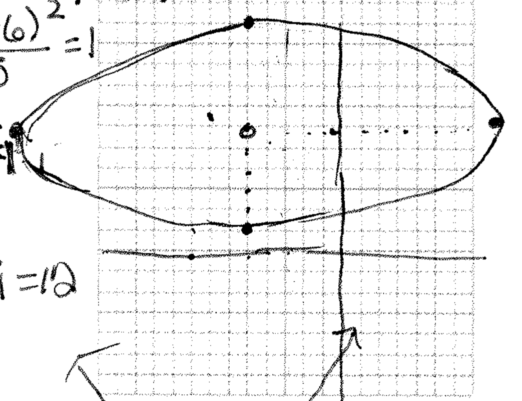
Co-Vertices: (-5, 11), and (-5, 1)

Foci: (7, 6), and (-17, 6)

Length of Major Axis: 26

Length of Minor Axis: 10

169 − 25 = 144   c = √144 = 12



23.

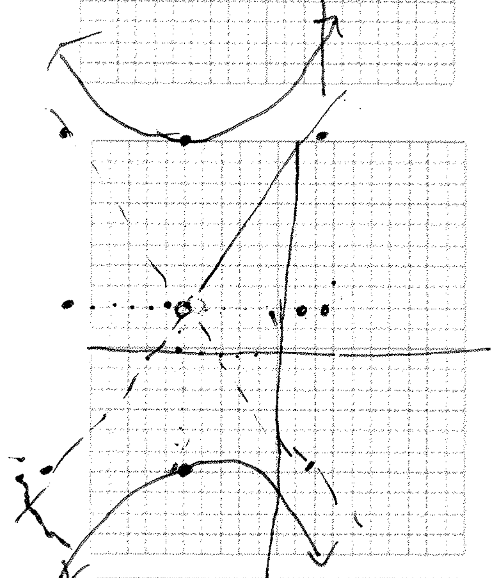
$$\frac{(y-2)^2}{64} - \frac{(x+5)^2}{36} = 1$$

Center: (-5, 2)

Vertices: (-5, 10), and (-5, -6)

Foci: ______, ______

Equations of Asymptotes: ____________



24.

$$4x^2 - 25y^2 + 72x + 50y + 199 = 0$$

$4(x^2+18x+81) - 25(y^2-2y+1) = -199 + 324 - 25$

$4(x+9)^2 - 25(y-1)^2 = 100$

$\frac{(x+9)^2}{25} - \frac{(y-1)^2}{4} = 1$

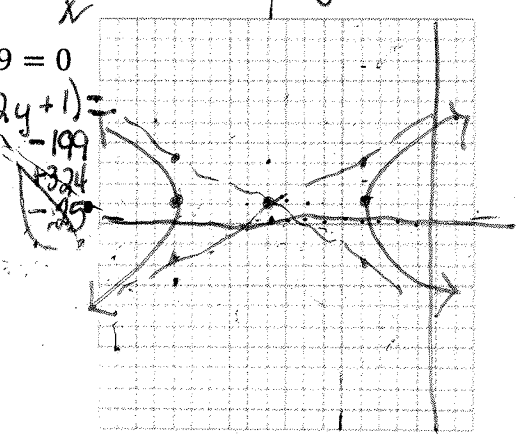
Center: (-9, 1)

Vertices: (-14, 1), and (-4, 1)

Foci: (-9, 1+√29) (-9, 1−√29)

Equations Asymptotes: $y-1 = \pm\frac{2}{5}(x+9)$

$c = \sqrt{25+4} = \sqrt{29}$



Solve

25. For a carnival, designers are planning a funhouse. They plan to put a large hyperbolic mirror inside this funhouse. They design the mirror's hyperbolic cross section on graph paper using a hyperbola with horizontal transverse axis. The asymptotes are to be $y = \frac{3}{2}x$ and $y = -\frac{3}{2}x$. They want the vertices to be 2 units from the origin. What equation should they use for the hyperbola?

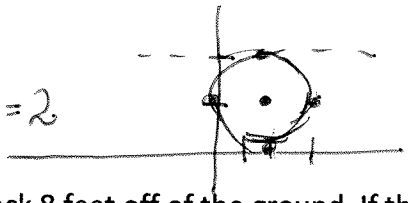
$\frac{x^2}{4} - \frac{y^2}{9} = 1$

26. Cory's hamster wheel sits on his desk, which is 3 feet off the ground. The equation for the wheel is $x^2 + y^2 - 4x - 2y = -4$. What is the maximum height Cory's hamster reaches on the wheel?

$x^2-4x+4+y^2-2y+1 = -4+4+1$

$(x-2)^2 + (y-1)^2 = 1$

$C=(2,1)$ $r=1$ $d=2$



5 ft off the ground

2 ft off desk

27. An ant sits on the minute hand of an analog clock 8 feet off of the ground. If the equation for the path of the tip of the minute hand is $x^2 + y^2 - 2x + 4y = 4$, what is the maximum height the ant will be at if he rides the minute hand around the clock?

$(x^2-2x+1) + (y^2+4y+4) = 4+1+4$

$(x-1)^2 + (y+2)^2 = 9$

$r=3$

$d=6$

8

14 ft

28. Sensor A is 2 km. north of sensor B. Sensor A detects the sound of an explosion 1.375 seconds after sensor B.

a) The speed of sound in air at the time of the explosion is 343.2 m/s. To the nearest meter, how much closer to the explosion is sensor B than sensor A? $(1.375)(343.2) \approx 472$ m

b) The location of the explosion lies on a hyperbola with foci at sensor A and sensor B. Let the point (0,0) represent the center of the hyperbola on a coordinate plane in units of kilometers. Write the equation of the hyperbola. Round values in the equation to the nearest thousandth.

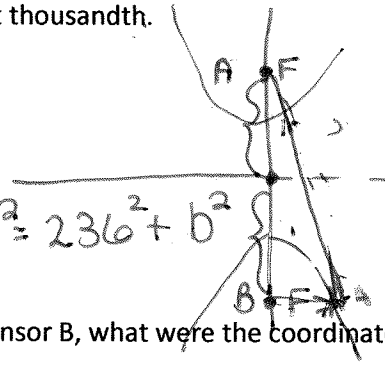
$472 = 2a$

$236 = a$

$1000 = c$

$c^2 = a^2 + b^2$

$1000^2 = 236^2 + b^2$



in meters

$\frac{y^2}{(236)^2} - \frac{x^2}{944304} = 1$

in km

$\frac{y^2}{.056} - \frac{x^2}{.944} = 1$

c) If the explosion was due east of sensor B, what were the coordinates of the explosion?

$-\frac{x^2}{944304} = 1 - \frac{(1000)^2}{(236)^2}$

$x^2 = 1601023.893$

$x \approx 1265.3$

$\frac{y^2}{(236)^2} - \frac{x^2}{944304} = 1$

$\frac{(-1000)^2}{(236)^2} - \frac{x^2}{944304} = 1$

in meters about $(1001, -1)$

in km $(3.99, -1)$

29 A doorway in a castle is shaped like a parabola. Find an equation describing the door given that is 4 feet across (on the ground) and 8 feet high in the center. Let the bottom left corner of the door be the origin.

vertex is (2, 8)
opens down

$(x-2)^2 = 4p(y-8)$

$(4-2)^2 = 4p(-8)$

$4 = -32p$

$-\frac{1}{8} = p$

$4p = -\frac{1}{2}$

$(x-2)^2 = -\frac{1}{2}(y-8)$

30. A fireplace arch is to be constructed in the shape of a semi-ellipse. The opening is to have a height of 2 feet at the center and a width of 6 feet along the base. The contractor draws the outline of the ellipse using tacks.

$c = \sqrt{9-4} = \sqrt{5}$

a) Find the equation of the full ellipse.

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

b) Give the required positions of the tacks.

$(\pm\sqrt{5}, 0)$

c) Give the length of the string the contractor would have to use to draw the outline.

$2a = 2(3) = 6$ ft