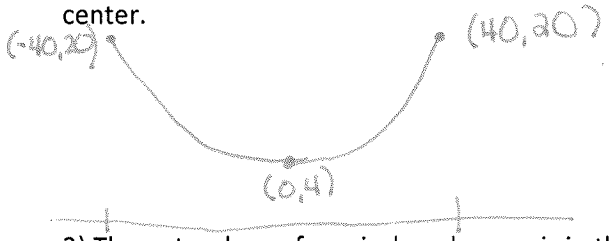
Honors Precalculus
Conics – Applications

NAME Key

1) The main cables of a suspension bridge are 20 meters above the road at the towers and 4 meters above the road at the center. The road is 80 meters long. Vertical cables are spaced every 10 meters. The main cables hang in the shape of a parabola. Find the equation of the parabola. Then, determine how high the main cable is 20 meters from the center.



$x^2 = 4p(y-4)$ $p = 100$ 8 m at 20 m from center

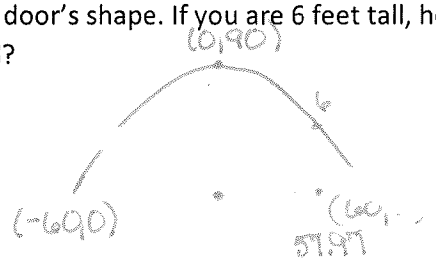
$(40)^2 = 4p(16)$ $x^2 = 100(y-4)$

$\frac{(40)^2}{16} = 4p$ $(20)^2 = 100(y-4)$

$400/100 = y-4$ $4+4=y$ $8=y$

2) The outer door of an airplane hangar is in the shape of a parabola. The door is 120 feet across and 90 feet high. Find the equation describing the door's shape. If you are 6 feet tall, how far must you stand from the edge of the door to keep from hitting your head?

about 2.03 ft from the edge



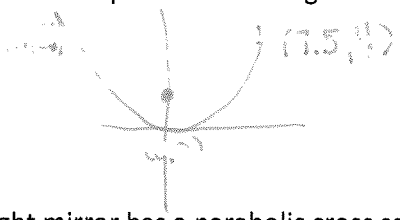
$x^2 = 4p(y-90)$ $p = -10$

$3600 = 4p(-90)$ $x^2 = -40(y-90)$

$\frac{3600}{-90} = p$ $x^2 = -40(0-90)$

$x^2 = 3600$ $x = 60$ 57.97

3) An engineer designs a satellite dish with a parabolic cross-section. The dish is 15 ft. wide at the opening and the depth is 4 feet. Find the position of the light source (the focus). How far is it from the deepest part of the dish?

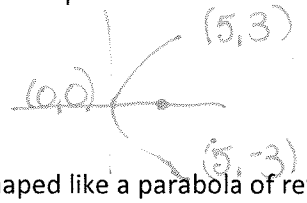


$x^2 = 4py$ $p = 225/64$

$(7.5)^2 = 4p(4)$ Locate at $(0, 225/64)$

$(7.5)^2/16 = p$ about $(225/64)$ ft or 3.52 ft

4) A car headlight mirror has a parabolic cross section with diameter of 6 in. and a depth of 5 in. How far from the vertex should the bulb be positioned if it is to be placed at the focus?

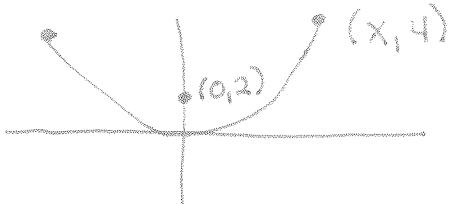


$x^2 = 4py$ $p = 9/20$

$(5)^2 = 4p(5)$ $9/20$ ths of an inch

$5/4(5) = p$ at $(9/20, 0)$

5) A searchlight is shaped like a parabola of revolution. If the light source is located 2 feet from the base along the axis of symmetry and the depth of the searchlight is 4 feet, what would the width of the opening be?



$x^2 = 4py$

$x^2 = 4 \cdot 2 \cdot 4$

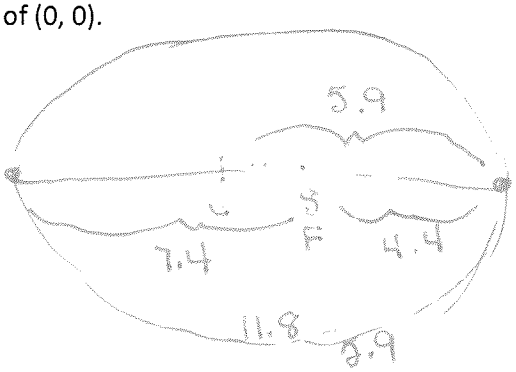
$x^2 = 32$

$x = \sqrt{32} = 4\sqrt{2}$

width = $2(4\sqrt{2}) = 8\sqrt{2}$ ft

about 11.31 ft

6) According to Kepler's Laws, planets have elliptical orbits, with the sun as one of the foci. The farthest Pluto gets from the sun is 7.4 billion kilometers. The closest it gets to the sun is 4.4 billion kilometers. Find the equation of Pluto's orbit assuming a center of (0, 0).



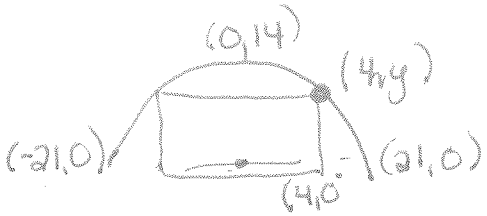
$\frac{x^2}{5.9^2} + \frac{y^2}{32.56} = 1$

$\frac{x^2}{34.81} + \frac{y^2}{32.56} = 1$

7) An elliptically shaped garden is surrounded by a wood walkway. The garden is 15 meters long and 8 meters wide. The walkway is 2 meters wide. Find the equation describing the ellipse that includes both the garden and the walkway.

$\frac{x^2}{(9.5)^2} + \frac{y^2}{6^2} = 1$ $\frac{x^2}{90.25} + \frac{y^2}{36} = 1$

8) An arch of a bridge over a highway is semi-elliptical in shape and 42 ft. across. The highest point of the arch is 14 feet above the highway. What is the maximum height, to the nearest inch, of a truck 8 ft. wide that can fit under the the arch? (assume the highway is one lane)



$\frac{x^2}{21^2} + \frac{y^2}{(14)^2} = 1$ $\frac{.74}{100} = \frac{x}{12}$

$\frac{4^2}{21^2} + \frac{y^2}{14^2} = 1$ $y = 13.74$ ft

about 13ft 9in

9) Suppose that two people standing 2 miles apart both see the burst from a fireworks display. After a period of time the first person, standing at point A hears the burst. One second later the second person, standing at point B hears the burst. If the person at point B is due west of the person at point A, and if the display is known to occur due north of the person at point A, where did the fireworks display occur?

Sound travels at 1100 ft/sec

person at A is 1100 ft closer than person at B.

difference of (x,y) to B (constant) and (x,y) to A is 1100 so (x,y) is on hyperbola with foci A,B

$\frac{x^2}{(550)^2} - \frac{y^2}{b^2} = 1$

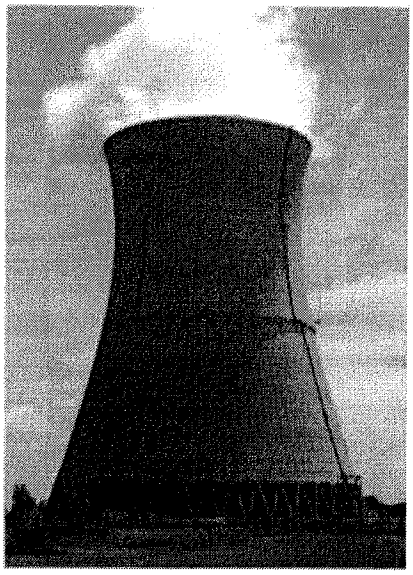
2a is the constant diff.

$2a = 1100$

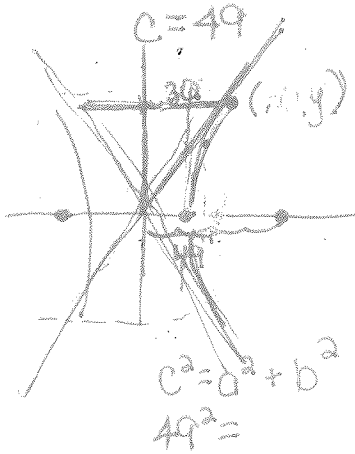
$a = 550$

$c = 5280$

10) A nuclear cooling tower cross section is a hyperbola having diameter of 60 ft. at the center. The distance between the two foci is 98 ft. What is the equation for the hyperbola?



$\frac{49}{2\sqrt{98}}$



$\frac{x^2}{30^2} - \frac{y^2}{1501} = 1$

$30^2 + b^2 = 49^2$

$\frac{x^2}{900} - \frac{y^2}{1501} = 1$

$c^2 = a^2 + b^2$

$49^2 =$

$\frac{x^2}{(550)^2} - \frac{y^2}{275\,75900} = 1$

$\frac{(5280)^2}{(550)^2} - \frac{y^2}{275\,75900} = 1$

$\frac{(5280)^2}{550^2} - 1 = \frac{y^2}{275\,75900}$

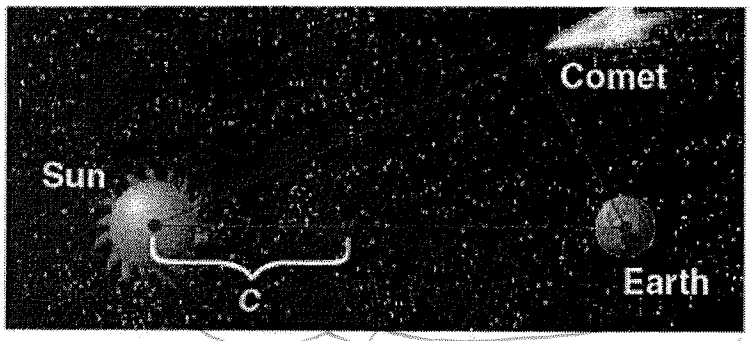
$y = 50138$ ft

about 9.49 miles North of the person at A

NOTE: Hyperboloid cooling towers have become the design standard for all natural-draft cooling towers because of their structural strength and minimum usage of material. The hyperboloid shape also aids in accelerating the upward convective air flow, improving cooling efficiency!

In the equation for any parabola $2a$ is the constant difference

11. Earth and Sun are 146 million kilometers apart. A comet follows a path that is one branch of a hyperbola. Suppose the comet is 30 million miles farther from the Sun than from the Earth. Determine the equation of the hyperbola centered at the origin for the path of the comet.



Find the center and "a" and "b"

The common difference is 30 (million)

$2a = 30$

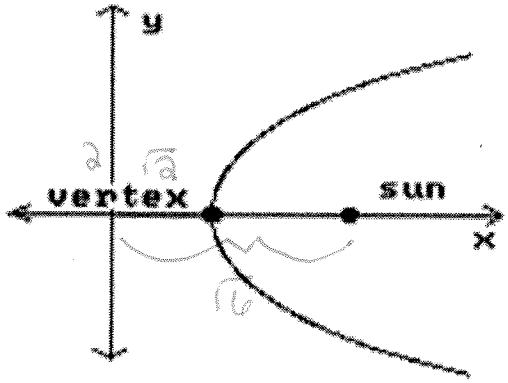
$a = 15$ (million)

$\frac{x^2}{225} - \frac{y^2}{5104} = 1$

$73^2 = 225 + b^2$

F C $\frac{146}{2} = 73$ (million km) F

12. A comet follows the hyperbolic path described by $\frac{x^2}{2} - \frac{y^2}{4} = 1$ where x and y are in millions of miles. If the sun is the focus of the path, how close to the sun is the vertex of the path?



$c = \sqrt{2+4} = \sqrt{6}$

$\sqrt{6} - \sqrt{2}$

≈ 1.04 millions of miles

13. The delivery area for a particular grocery store can be represented by a circle, and extends to the points (0, 20) and (-5, 12). These points are on the diameter of the circle. Find the equation of the circle. What is the area of the delivery area being covered by this particular store?

$d = \sqrt{64 + 25}$

$= \sqrt{99}$

$r = \sqrt{99}/2$

$r^2 = 99/4$

(-5,12) (0,20) C(-5/2, 16)

$(x + 5/2)^2 + (y - 16)^2 = 99/4$

$(x + 2.5)^2 + (y - 16)^2 = 24.75$