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.1. Write each of the Pythagorean identities in at least 3 different equivalent equations.

$\sin^2x+\cos^2x=1$ $\cos^2x=1-\sin^2x$ $-\cos^2x=\sin^2x-1$ $\sin^2x=1-\cos^2x$ $-\sin^2x=\cos^2x-1$	$\tan^2x+1=\sec^2x$ $1=\sec^2x-\tan^2x$ $\tan^2x-\sec^2x=-1$	$\cot^2x+1=\csc^2x$ $\cot^2x-\csc^2x=-1$ $1=\csc^2x-\cot^2x$

2. Which of the following expressions in NOT equivalent to the other three?	a) $\frac{\cos x}{\sin x}$ b) $\cot x$ c) $\frac{1}{\tan x}$ d) $1+\cot^2 x$

3. **Simplify the following trig expressions:**

a) $\frac{sec^2x-1}{secx+1}=\frac{\cancel{(secx+1)}(secx-1)}{\cancel{(secx+1)}}$ $=secx-1$	b) $\frac{cos^2x-1}{cos^2x}=\frac{-\sin^2x}{\cos^2x}$ $=-\tan^2x$	c) $\underline{csc^2x-cot^2x}+sec^2x-1$ $=1+\sec^2x-1$ $=\sec^2x$
d) $\frac{cos^2x-1}{sin^2x-1}=\frac{-\sin^2x}{-\cos^2x}=\tan^2x$	e) $\frac{1+\cot^2 x}{\sec^2 x}$ $\frac{\csc^2x}{\sec^2x}=$ $=\frac{\frac{1}{\sin^2x}}{\frac{1}{\cos^2x}}$ $=\frac{\cos^2x}{\sin^2x}=\cot^2x$	f) $\cos x\,(\sec x+\tan x)$ $\cos x\left(\frac{1}{\cos x}+\frac{\sin x}{\cos x}\right)$ $1+\sin x$

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4. Verify the following trig identities:

a) $\frac{tanx+cotx}{cotx} = sec^2x$

$\frac{tanx}{cotx} + \frac{cotx}{cotx} = sec^2x$

$\frac{\frac{tanx}{1}}{\frac{1}{tanx}} + 1 = sec^2x$

$tan^2x + 1 = sec^2x$

$sec^2x = sec^2x$

b) $\frac{1}{secx-tanx} - \frac{1}{secx+tanx} = 2tanx$

$(secx+tanx)-(secx-tanx) = 2tanx$

$\frac{2tanx}{sec^2x-tan^2x} = 2tanx$

$\frac{2tanx}{1} = 2tanx$

$2tanx = 2tanx$

c) $\cos\left(\frac{3\pi}{2} + x\right) = sinx$

$\cos\frac{3\pi}{2}cosx - \sin\frac{3\pi}{2}sinx = sinx$

$(0)(cosx) - (-1)(sinx) = sinx$

$sinx = sinx$

5. Verify the following:

a) cos (x + 30) + sin (x – 60) = 0

$cosx\cos30 - sinx\sin30 + sinx\cos60 - \sin60cosx$

$\frac{\sqrt{3}}{2}cosx - \frac{1}{2}sinx + \frac{1}{2}sinx - \frac{\sqrt{3}}{2}cosx = 0$

$0 = 0$

b) cos (x + y) + cos (x – y) = 2 cos x cos y

$cosxcosy - sinxsiny + cosxcosy + sinxsiny$

$2cosxcosy = 2cosxcosy$

c) $\cos^2 5x - \sin^2 5x = \cos 10x$

$= \cos(2)(5x)$

$= \cos^2 5x - \sin^2 5x$

d) sin 4x = 4 cos x sin x cos 2x

$\sin 2(2x)$

$2\sin2x\cos2x$

$2 \cdot 2 sinx cosx \cos2x$

$4 sinx cosx \cos2x$

e) sin 6x = 2 sin 3x cos 3x

$\sin 2(3x)$

$2\sin3x\cos3x$

f) $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$\frac{1}{2}(1 + 2\cos^2x - 1)$

$\frac{1}{2}(2\cos^2x)$

$\cos^2x$

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6. First, find the solutions of the equations on the interval $[0, 2\pi)$ then write the general solution for each equation. When possible, find exact values. When not possible, round answers to three decimal places.

a) $sin^2x - 1 = 0$ $\sin^2 x = 1$ $\sin x = \pm 1$	**b)** $2cos^2x + cosx = 0$ $\cos x(2\cos x + 1) = 0$ $\cos x = 0 \quad \cos x = -\frac{1}{2}$	**c)** $2sin^2x - sinx - 1 = 0$ $(2\sin x + 1)(\sin x - 1) = 0$ $\sin x = -1/2 \quad \sin x = 1$
Interval solution: $x = \pi/2, 3\pi/2$	Interval solution: $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$	Interval solution: $x = 7\pi/6, 11\pi/6, \pi/2$
General solution: $x = \frac{\pi}{2} + \pi K;\ K \in \mathbb{Z}$	General solution: $x = \frac{\pi}{2} + \pi K,$ $\frac{2\pi}{3} + 2\pi K,\ K \in \mathbb{Z}$ $\frac{4\pi}{3} + 2\pi K$	General solution: $x = \frac{7\pi}{6} + 2\pi K,$ $\frac{11\pi}{6} + 2\pi K,\ K \in \mathbb{Z}$ $\frac{\pi}{2} + 2\pi K$

7. First, find the solutions of the equations on the interval $[0, 360°)$ then write the general solution for each equation. When possible, find exact values. When not possible, round answers to three decimal

a) $4sin^2x - 3 = 0$ $\sin^2 x = \frac{3}{4}$ $\sin x = \pm\frac{\sqrt{3}}{2}$	**b)** $\cos 2x + 3\cos x + 2 = 0$ $2\cos^2 x - 1 + 3\cos x + 2 = 0$ $2\cos^2 x + 3\cos x + 1 = 0$ $(2\cos x + 1)(\cos x + 1) = 0$ $\cos x = -1/2 \quad \cos x = -1$	**d)** $\sin 2x = -\cos x$ $2\sin x\cos x + \cos x = 0$ $\cos x(2\sin x + 1) = 0$ $\cos x = 0 \quad \sin x = -1/2$
Interval solution: $60°, 120°, 240°, 300°$	Interval solution: $x = 120°, 240°, 180°$	Interval solution: $x = 90°, 270°, 210°, 330°$
General solution: $x = 60° + 180°K,$ $120° + 180°K$ $K \in \mathbb{Z}$	General solution: $x = 120° + 360°K,$ $240° + 360°K,$ $180° + 360°K$ $K \in \mathbb{Z}$	General solution: $x = 90° + 180°K,$ $210° + 360°K$ $330 + 360°K$

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8. Find the exact value of the following:

a) $\cos 112.5°$ $\cos\left(\frac{1}{2}\cdot 225\right) = -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$ $= -\frac{\sqrt{2-\sqrt{2}}}{2}$	b) $\sin\frac{11\pi}{12} = \sin 165°$ $\sin(120° + 45°) =$ $= \sin 120° \cos 45° + \cos 120° \sin 45°$ $= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{-1}{2}\cdot\frac{\sqrt{2}}{2}$ $= \frac{\sqrt{6}-\sqrt{2}}{4}$	c) $\tan\frac{3\pi}{8} = \tan 67.5$ $= \tan\frac{1}{2}(135°)$ $= \frac{1-\cos 135°}{\sin 135°} = \frac{1+\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$ $= \frac{2+\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}+2}{2} = \sqrt{2}+1$

9. Complete each statement:

a) If $\cos x = k$, then $\cos(-x) = K$	b) If $\tan x = a$, then $\tan(2\pi - x) = -a$ ~~K~~

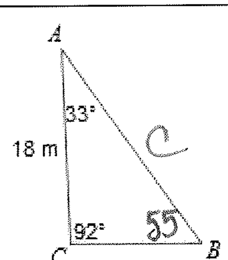
10. Given that $\sin x = \frac{3}{5}$, and x is in Q1, what is the value of the expression $\cos\left(\frac{\pi}{2} - x\right)$?

$\frac{3}{5}$ (cofunctions)

or $\cos\frac{\pi}{2}\cdot\cos x + \sin\frac{\pi}{2}\sin x$
$= (0)(\cos x) + (1)(3/5)$
$= 3/5$

11. Solve for all missing pieces of the following triangles:

a)

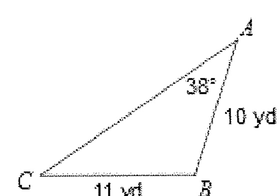


$\frac{\sin 92}{c} = \frac{\sin 55}{18} = \frac{\sin 33}{a}$

$c = \frac{18\sin 92°}{\sin 55} \approx 21.96$

$a = \frac{18\sin 33}{\sin 55} \approx 11.97$

b)



$\frac{\sin 38}{11} = \frac{\sin B}{b} = \frac{\sin C}{10}$

case 1
$\angle C \approx 34.03$
$\angle B \approx 107.97$
$b \approx 16.995$

case 2
~~145.97~~
~~+38~~
~~> 180~~

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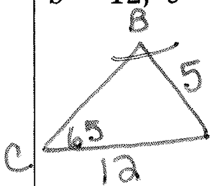
12. Solve each of the following using Law of Sines or Law of Cosines.

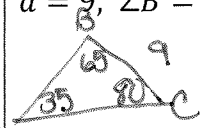
a) For ΔABC (draw it), $\angle C = 65°$, $b = 12$, $c = 5$. Find $\angle B$.



$\frac{\sin 65}{5} = \frac{\sin B}{12}$

no triangle possible

b) For ΔABC (draw it), $\angle A = 35°$, $a = 9$, $\angle B = 65°$. Find b.



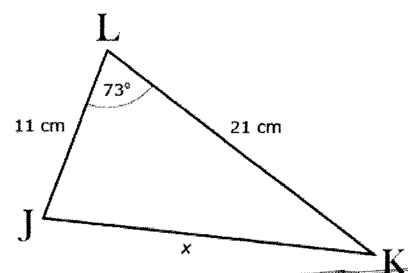
$\angle C = 80°$

$\frac{\sin 80}{c} = \frac{\sin 65}{b} = \frac{\sin 35}{9}$

* $b \approx 14.22$

$c \approx 15.45$

c) For the ΔJKL), find side length x.



$x = \sqrt{11^2 + 21^2 - 2(11)(21)\cos 73}$

$x \approx 20.662$

13. State the number of possible triangles given the following information, then solve the triangle(s).

a) $m\angle B = 89°, b = 24, a = 12$ one triangle

$\frac{\sin 89}{24} = \frac{\sin A}{12} = \frac{\sin C}{c}$

case 1

$\angle A \approx 29.995°$

$\angle C \approx 61.005°$

$c \approx 20.995$

~~case 2~~

~~$\angle A \approx 150.005$~~

~~$+ 89$~~

~~> 180~~

b) $m\angle A = 86°, c = 7, a = 4$

$\frac{\sin 86}{4} = \frac{\sin C}{7} = \frac{\sin B}{b}$

case 1

~~$\angle C =$~~

no triangle possible

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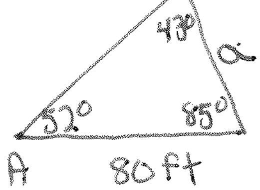
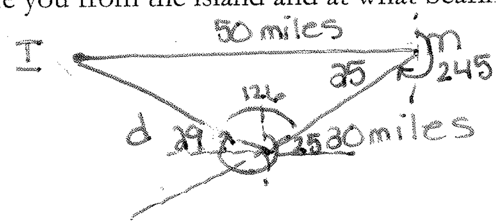
14. Joanna is interested in determining the height of a tree. She is at point A, 80 feet from the base of the tree, and she notices that the angle of elevation to the top of the tree is 52°. The tree is leaning towards her and is growing at an angle of 85° with respect to the ground. What is the height of the tree?



$\frac{\sin 43}{80} = \frac{\sin 52}{a}$ $a \approx 92.4$ ft

15. You are sailing a boat and need to get to an island that is 50 miles to the West of your current location. But there are rough seas along the direct route. To avoid this, you take a path at a bearing of 245 for 30 miles. How far away are you from the island and at what bearing must you now sail to get to the island?



$d = \sqrt{50^2 + 30^2 - 2(50)(30)\cos 225^\circ}$

$x \approx 26.1$ miles

$50^2 = 30^2 + 26.1^2 - 2(30)(26.1)\cos A$

$\angle A \approx 126^\circ$

$180 - 126 - 25$

$270^\circ + 29 = 299^\circ$

bearing is 299°

16. Which of the following statements is NOT true?

a) You can use the law of sines if you know any two angles and any one side of the triangle.

b) You can use the law of sines if you know the three sides of a triangle.

c) You can use the law of cosines if you know any two sides and any one angle of a triangle

d) You can use the law of cosines if you know the three sides of a triangle.

17. You CANNOT use the law of cosines if you know:

a) the three sides of a triangle

b) two sides and the included angle

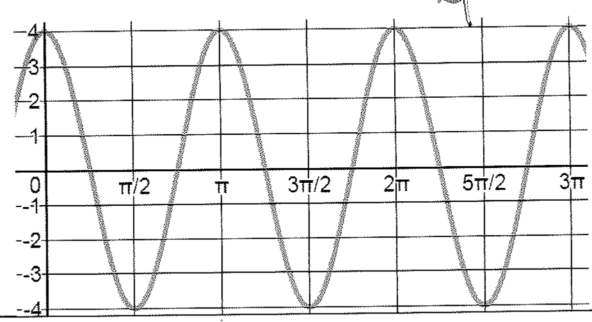
c) two angles and the included side

d) two sides and the non-included angle

18. How many solutions are thereto the following equations over the interval $[0, 2\pi]$

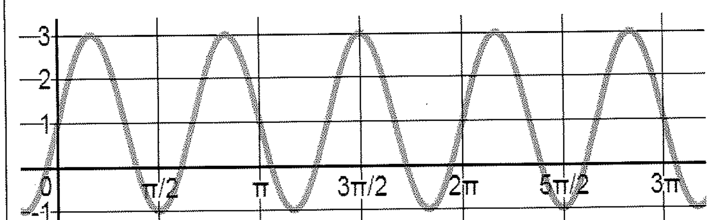
a) $4 \cos 2x = 3$

4



b) $2 \sin 3x + 1 = 2$

6



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18. The frequency of the musical note A_3 is 220 Hz. $y = a\sin(2\pi f t)$

a) What is the equation for a sine wave that represents A_3 played at a volume of 75 dB?

$a = 75$ $f = 220$
$y = 75\sin(440\pi t)$

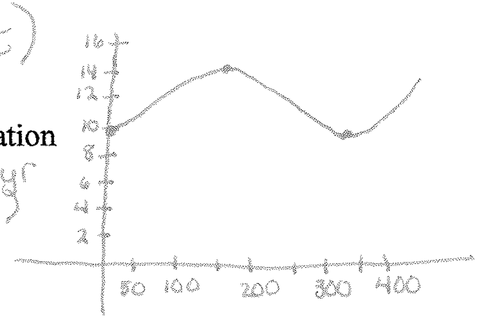
b) A note one octave higher vibrates twice as fast. What would the equation for the sine wave be if the note described above was played one octave higher?

$f = 440$ $a = 75$ $y = 75\sin(880\pi)$

c) What would the equation be if the note were played one octave lower?

$f = 110$ $a = 75$
$y = 75(220\pi t)$

19) The number of daylight hours for San Diego can be modeled by the equation $h(d) = 2.4\sin\left[\frac{2\pi}{365.25}(d - 80)\right] + 12$, for d = day of year. (days of yr table on p 265)



a) Graph the function.

b) On what dates does San Diego have 10 hours of daylight? January 22, November 16

c) On what dates does San Diego have more than 15 hours of daylight? never

d) Rounded to the nearest hour, what are the maximum and minimum numbers of hours of daylight in San Diego? 10 is the minimum and 14 is the maximum

20) For ΔABC, $b = 7.5$, $c = 9.8$, and $A = 50.5°$. Which equation can be used to calculate the value of of side a?

A) $a = \sqrt{(7.5)^2 + (9.8)^2 + (2)(7.5)(9.8)\cos 50.5°}$

B) $a = \sqrt{(7.5)^2 + (9.8)^2 - (2)(7.5)(9.8)\cos 50.5°}$

C) $a = \sqrt{(7.5)^2 + (9.8)^2 + (7.5)(9.8)\cos 50.5°}$

D) $a = \sqrt{(7.5)^2 + (9.8)^2 - (7.5)(9.8)\cos 50.5°}$