Feb 22

HONORS PRECALC REVIEW
Sum and Difference Identities
Double and Half Angle Identities

NAME Key

Period______

Directions: Tell whether each statement is true or false.

1) $\sin 75 = \sin 50 \cos 25 - \cos 25 \sin 25$

False

$\sin(75) = \sin(50 + 25)$

$= \sin 50 \cos 25 + \cos 25 \sin 25$

2) $\cos 15 = \cos 60 \cos 45 + \sin 60 \sin 45$

True

$\cos(60° - 45°)$

3) $\tan 225 = \dfrac{\tan 180 - \tan 45}{1 + \tan 180 \tan 45}$

False

$\tan 225 = \tan(180 + 45)$

$= \dfrac{\tan 180° + \tan 45°}{1 - \tan 180° \tan 45°}$

Directions: Write the expression as the sine, cosine or tangent of an angle.

4) $\sin 42 \cos 17 - \cos 42 \sin 17$

$\sin(42 - 17) = \sin(25°)$

5) $\dfrac{\tan 19 + \tan 47}{1 - \tan 19 \tan 47}$

$\tan(19° + 47°)$

$= \tan(66°)$

6) $\cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$\cos(\pi/3 - \pi/4) =$

$\cos(\pi/12)$

Directions: Use the sum or difference identity to find the exact value.

7) $\tan 195° = \tan(150 + 45°)$

$= \dfrac{\tan 150° + \tan 45°}{1 - \tan 150° \tan 45}$

$= \dfrac{-\frac{\sqrt{3}}{3} + 1}{1 - \sqrt{3}/3} = \dfrac{-\sqrt{3} + 3}{3 - \sqrt{3}} = \dfrac{(\sqrt{3} + 3)(\sqrt{3} + 3)}{(3 - \sqrt{3})(3 + \sqrt{3})}$

$= \dfrac{3 + 6\sqrt{3} + 9}{6}$

$= \dfrac{12 - 6\sqrt{3}}{6}$

$2 - \sqrt{3}$

8) $\cos 255° = \cos(300° - 45°) =$

$\cos 300° \cos 45° + \sin 300° \sin 45°$

$-\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + -\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}$

$= \dfrac{\sqrt{2} - \sqrt{6}}{4}$

9) $\sin 165°$

$\sin(120° + 45°)$

$\sin(120°)\cos 45° + \cos 120 \sin 45°$

$\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + -\frac{1}{2}\frac{\sqrt{2}}{2}$

$\dfrac{\sqrt{6} - \sqrt{2}}{4}$

10) $\cos\frac{13\pi}{12} = \cos 195°$

$\frac{13\pi}{12} \cdot \frac{180}{\pi}$

$\cos(135° + 60°) =$

$\cos 135° \cos 60° - \sin 135° \sin 60°$

$-\frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}$

$\dfrac{-\sqrt{2} - \sqrt{6}}{4}$

13) $\sin(\alpha - \beta)$

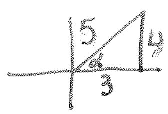
Given: $\cos\alpha = \frac{3}{5}$, where $0 < \alpha < \frac{\pi}{2}$

$\tan\beta = \frac{12}{5}$, where $0 < \beta < \frac{\pi}{2}$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\left(\frac{4}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$

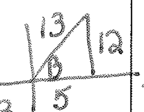
$\dfrac{20 - 36}{65} = \dfrac{-16}{65}$



$\sin\alpha = 4/5$

$\sin\beta = 12/13$

$\cos\beta = 5/13$



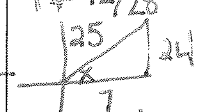
14) $\tan(x - y)$

Given: $\cos x = \frac{7}{25}$, where $0° < x < 90°$

$\cos y = -\frac{4}{5}$, where $90° < y < 180°$

$\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

$\dfrac{\frac{24}{7} + \frac{3}{4}}{1 + 72/28} = \dfrac{96 + 21}{-44} = \dfrac{117}{-44}$



$\tan x = 24/7 \quad \tan y = -3/4$

15) $\sin(\alpha + \beta)$

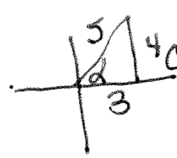
Given: $\sin\alpha = \frac{4}{5}$, where α is in Quadrant I

$\cos\beta = -\frac{24}{25}$, where β is in Quadrant III

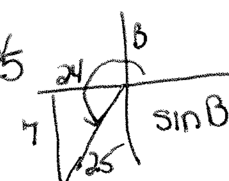
15. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\frac{4}{5}\right)\left(\frac{-24}{25}\right) + \left(\frac{3}{5}\right)\left(\frac{-7}{25}\right)$

$-\frac{96}{125} - \frac{21}{125} = \boxed{\frac{-117}{125}}$



$\cos\alpha = 3/5$



$\sin\beta = -7/25$

16) $\cos(x + y)$

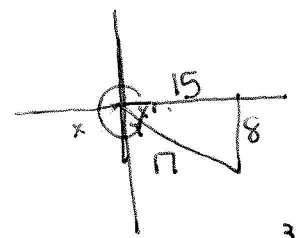
Given: $\cos x = \frac{15}{17}$, where $\frac{3\pi}{2} < x < 2\pi$

$\tan y = \frac{4}{3}$, where $\pi < y < \frac{3\pi}{2}$

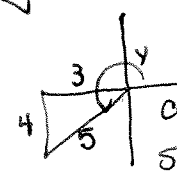
$\cos(x + y) = \cos x \cos y - \sin x \sin y$

$\frac{15}{17} \cdot \frac{-3}{5} - \frac{-8}{17} \cdot \frac{-4}{5}$

$\frac{-45}{85} - \frac{32}{85} = \frac{-77}{85}$



$\sin x = -8/17$



$\cos y = -3/5$

$\sin y = -4/5$

Directions: Tell whether each statement is true.

1) $\cos 2(20°) = 2\cos^2 40° - 1$

False

it would be $2\cos^2(20°) - 1$

2) $\cos(70°) = \cos^2 35° - \sin^2 35°$

True

$\cos(2 \cdot 35)$

3) $\tan\frac{140°}{2} = -\sqrt{\frac{1-\cos 140°}{1+\cos 140°}}$

QI

False

it would be pos

Directions: Find the exact value of the given function.

4) $\cos 75°$

$\cos\left(\frac{1}{2}(150°)\right) = \sqrt{\frac{1+\cos 150}{2}}$

$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$

5) $\sin\frac{5\pi}{8}, \frac{180}{\pi} = \sin 112.5$

$\sin\left(\frac{1}{2} \cdot 225\right) = \sqrt{\frac{1-\cos 225}{2}}$

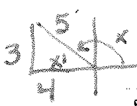
$= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2+\sqrt{2}}}{2}$

Directions: For #6-9: If $\sin x = \frac{3}{5}$ and x is in Quadrant II, find each value. Draw the reference triangle.

6) $\cos 2x$

$\cos 2x = \cos^2 x - \sin^2 x$

$(16/25) - (9/25) = 7/25$



$\cos x = ^-4/5$

$\tan x = ^-3/4$

7) $\tan 2x$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$= \frac{2(-3/4)}{1-(-3/4)^2} = \frac{-\frac{3}{2}}{1-\frac{9}{16}} = \frac{-\frac{3}{2}}{\frac{7}{16}} =$

$-\frac{3}{2}\left(\frac{16}{7}\right) = \boxed{\frac{-24}{7}}$

8) $\sin\frac{x}{2}$

$\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1-\cos x}{2}}$

$= \sqrt{\frac{1+4/5}{2}}$

$= \sqrt{\frac{\frac{9}{5}}{2}}$

$= \sqrt{\frac{9}{10}}$

$= \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$

9) $\cos\frac{x}{2}$

$\cos\left(\frac{x}{2}\right) = \sqrt{\frac{1+\cos x}{2}}$

$= \sqrt{\frac{1+{-4/5}}{2}}$

$= \sqrt{\frac{\frac{1}{5}}{2}}$

$= \sqrt{\frac{1}{10}}$

$= \boxed{\frac{\sqrt{10}}{10}}$

Directions: For #10-13: If $\cos\theta = -\frac{1}{3}$ and x is in Quadrant II, find each value. Draw the reference triangle.

10) $\cos 2\theta$

$= \cos^2\theta - \sin^2\theta$

$(-1/3)^2 - \left(\frac{2\sqrt{2}}{3}\right)^2 = 1/9 - 8/9 = \boxed{\frac{-7}{9}}$

$\sin\theta = \frac{2\sqrt{2}}{3}$

11) $\sin 2\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{2\sqrt{2}}{3}\right)\left(\frac{-1}{3}\right)$

$= \boxed{\frac{-4\sqrt{2}}{9}}$

12) $\tan\frac{\theta}{2}$

$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta}$

$= \frac{1-\frac{-1}{3}}{\frac{2\sqrt{2}}{3}} = \frac{3+1}{2\sqrt{2}}$

$= \frac{4}{2\sqrt{2}}$

$= \frac{2}{\sqrt{2}}$

$= \boxed{\sqrt{2}}$

13) $\sin\frac{\theta}{2}$

$\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$

$= \sqrt{\frac{1+\frac{1}{3}}{2}}$

$= \sqrt{\frac{\frac{4}{3}}{2}}$

$= \sqrt{2/3} = \boxed{\frac{\sqrt{6}}{3}}$