

Honors Precalculus Semester 1 Review

Name: _____

UNIT 1

1. For each sequence, find the explicit **and** recursive formulas. Show your work.

<p>a) 45, 39, 33, 27 ...</p> <p>Explicit formula:</p> <p>Recursive formula:</p>	<p>b) $-\frac{8}{3}, -\frac{16}{9}, -\frac{32}{27}, -\frac{64}{81}, \dots$</p> <p>Explicit formula:</p> <p>Recursive formula:</p>
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2. Given the recursive formula for each sequence, find the explicit formula.

<p>a) $\begin{cases} a_1 = 9 \\ a_n = a_{n-1} - 8 \end{cases}$</p> <p>Explicit formula:</p>	<p>b) $\begin{cases} a_1 = 5 \\ a_n = -\frac{1}{2}(a_{n-1}) \end{cases}$</p> <p>Explicit formula:</p>
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3. Given the sequence, find the term named for each problem.

<p>a) 8, 12, 18, ...</p> <p>Find the 10th term.</p>	<p>b) 8, 10, 12, ...</p> <p>Find the 42nd term.</p>	<p>c) $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \dots$</p> <p>Find a_{13}</p>
<p>d) the eighth term of the sequence</p> <p style="text-align: center;">$3, 3\sqrt{6}, 18, 18\sqrt{6}, \dots$</p>	<p>e) a_6 of the sequence</p> <p style="text-align: center;">$6, 7.2, 8.64, 10.368, \dots$</p>	

4. Solve each of the following:

<p>a) Find x such that $x - 4$, x, and $3x - 8$ are three consecutive terms of a geometric sequence.</p>	<p>c) Find x such that $x + 4$, $3x - 9$, and $2x + 8$ are consecutive terms of an arithmetic sequence.</p>
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5. Evaluate each of the following:

<p>a)</p> $\sum_{n=3}^7 \left(81 \left(\frac{1}{3} \right)^{n-1} \right)$	<p>b)</p> $\sum_{n=1}^5 (3n + 4)$	<p>c)</p> $\sum_{j=0}^5 \left(\frac{1}{2} \right)^j$
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6. Answer each of the following:

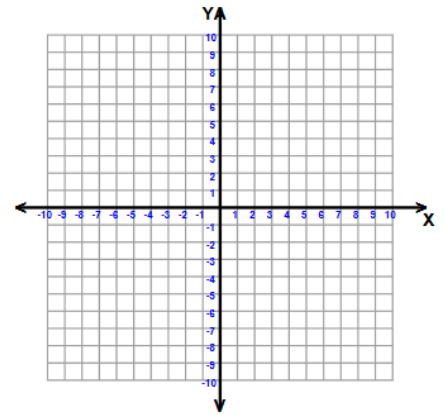
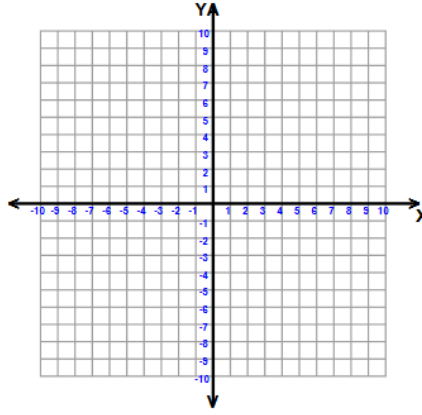
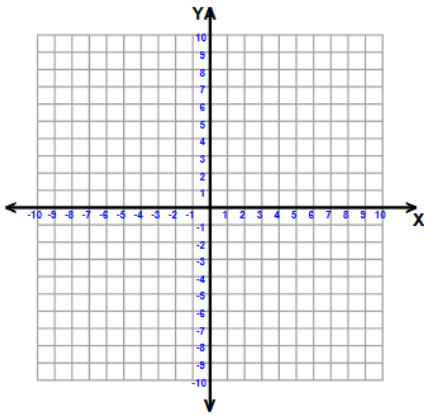
<p>a) The sum of a geometric series of 5 terms is 60.5 The first term is $\frac{1}{2}$ and $r = 3$. Write the series using sigma notation.</p>	<p>b) i) Express the arithmetic series below using sigma notation.</p> $7 + 18 + 29 + 40 + \dots$ <p>ii) Find the sum of the first 20 terms.</p>
<p>c) The front row of a theater has 20 seats. Each of the rows behind it has two more seats than the row before it.</p> <p>i) Represent the situation using sigma notation.</p> <p>ii) How many total seats are there in the first 25 rows?</p>	<p>d) If \$2000 is deposited in an account that pays 5% interest compounded monthly, how much will be in the account after 20 years? (nearest cent)</p>
<p>e) The population of a bacteria colony is growing at a rate of 10% per hour. How many hours will it take until the population is 4 times its original size? (nearest hundredth)</p>	<p>f) \$2000 is invested in an account earning 5% annual interest compounded continuously. How long will it take for the account balance to reach \$5000? (nearest hundredth of a year)</p>

7. Graph each of the following and describe the transformations that are occurring to the exponential function.

a. $y = -.5(4)^x$

b. $y = 3^{x+1} - 3$

c. $y = 3(2)^{-x}$



8. Condense each logarithmic expression using the properties of logarithms. (write as a single logarithm)

a) $2 \log 4 + \log 7 - 3 \log 5$	b) $3 \ln x - \ln 4x + .5 \ln(x - 1)$	c) $\log 2x + \frac{1}{3} \log 125 - 2 \log 5x$
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9. Expand each of the following using the properties of logarithms.

a) $\log \left(\frac{3x^2}{y-2} \right)$	b) $\log_2(x^3 y^2)^3$	c) $\ln \left(\frac{x^4 \sqrt{y}}{z} \right)$
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10. Solve for x (round answer to the nearest thousandth if necessary):

a) $\log_3(2x) - \log_3 4 = 3$	b) $\log_7 x + \log_7(x - 1) = \log_7 6$	c) $9^{4x-1} = 7$
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11. Determine if each function is even, odd, or neither by checking to see if $f(x) = f(-x)$ or $f(-x) = -f(x)$. Then determine the symmetry as well.

a) $f(x) = x^3 - 3x$	b) $g(x) = x^4 - 5x^2 + 1$
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12. Which of the following functions are even?

a) $f(x) = x^2$	b) $f(x) = -3x^2$	c) $f(x) = x^2 - 4$	d) $f(x) = (x - 7)^2$
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13. Which of the following functions are odd?

a) $f(x) = x^3$	b) $f(x) = 3x^3$	c) $f(x) = x^3 - 4$	d) $f(x) = (x - 7)^3$
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14. Find $f(g(x))$ and $g(f(x))$ for the following:

a) $f(x) = 2x + 5$ and $g(x) = \frac{3}{x-4}$	b) $f(x) = 4x - 1$ and $g(x) = -2x^2$
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15. Write the inverse for the following, include any restrictions on the domain of the inverse.

a) $f(x) = 5x^2 + 1; x \geq 0$	b) $f(x) = 3\sqrt{2-x} - 5$
c) $f(x) = \sqrt[3]{x-6}$	d) $y = \log_3(x-1)$
e) $f(x) = 2(3)^x - 1$	

16. Describe all the transformations that occur from the given parent function $f(x)$ to the transformed function $g(x)$.

<p>a) $f(x) = x^2$; $g(x) = -2(x + 4)^2 - 3$</p>	<p>b) $f(x) = \sqrt{x}$; $g(x) = .25\sqrt{3-x} + 4$</p>
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17) Write the equation for $g(x)$.

<p>a) The graph $g(x)$ is a translation 2 units to the right of the graph $f(x) = e^x$.</p>	<p>b) The graph $g(x)$ is a vertical stretch of the graph $f(x) = x^2 + 4x$ by a factor of 2 followed by a translation 4 units down.</p>
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UNIT 2

18). A student collects data and records it in the tables shown. Which type of function could be used to model the data? (Linear, Quadratic, Cubic, Exponential)

a) _____

Year	1980	1982	1985	1990	1995	1998	2000	2003	2004	2005
Price	3.68	5.17	6.12	5.8	6.06	6.82	7.76	9.52	10.74	13.84

b) _____

T	1	2	3	4	5	6	7	8	9	10
Y	1.6	2.7	4.4	6.4	8.9	13.1	19.3	28.2	38.2	48.7

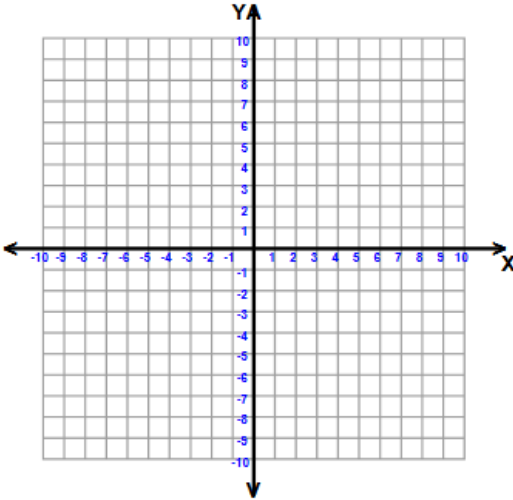
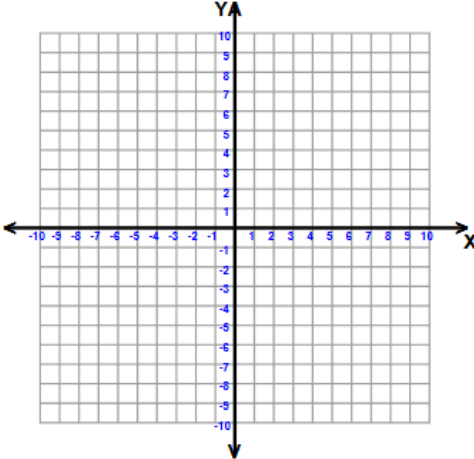
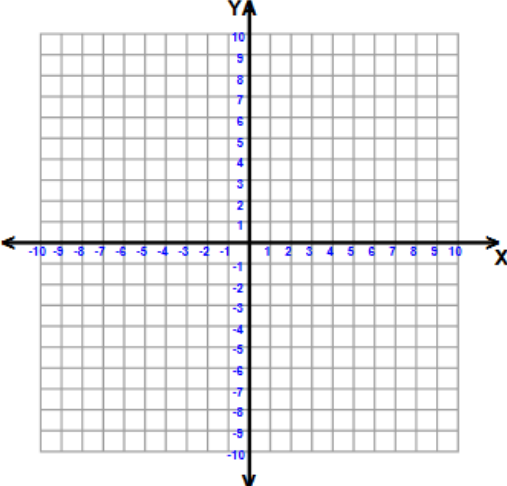
c) _____

Angle	10°	15°	20°	24°	30°	34°	40°	45°	48°	50°	58°	60°	64°
Distance	115.6	157.2	189.2	220.8	253.8	269.2	284.8	285.0	277.4	269.2	244.2	231.4	180.4

d) _____

Weight	19	19.5	20	20.5	21	22	22.5	23	23.5	24
Height	10.35	10.3	10.25	10.2	10.1	9.85	9.8	9.79	9.7	9.6

19. Sketch a graph of each of the following functions. List the key characteristics.

<p>a) $f(x) = (x + 1)(x - 1)^2$</p> <p>Domain:</p> <p>Range:</p> <p>End Behavior:</p> <p>Zeros:</p> <p>y-intercept:</p> <p>Maxima:</p> <p>Minima:</p>	
<p>b) $f(x) = -(x - 2)(x + 2)(x - 4)$</p> <p>Domain:</p> <p>Range:</p> <p>End Behavior:</p> <p>Zeros:</p> <p>y-intercept:</p> <p>Maxima:</p> <p>Minima:</p>	
<p>c) $f(x) = x^4 + 2x^3$</p> <p>Domain:</p> <p>End Behavior:</p> <p>Zeros:</p> <p>y-intercept:</p> <p>Maxima:</p> <p>Minima:</p>	

d) $f(x) = -2x^4 + 2x^2 - 2$

Domain:

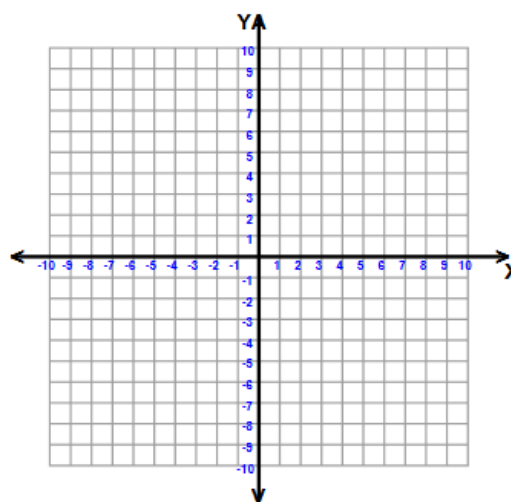
End Behavior:

Zeros:

y-intercept:

Maxima:

Minima:



20. List the possible rational zeros for each function.

a) $f(x) = 3x^5 + 16x^4 - 2x^2 + 5x - 18$

b) $f(x) = -7x^3 - 3x^2 + 18x + 35$

21. Use Descartes Rule to describe the roots of each of the following:

a) $f(x) = x^4 + 3x^3 - 2x^2 - 4x + 6$

b) $f(x) = -3x^3 + 2x^2 - x + 7$

22. Describe the end behavior for each function.

a) $f(x) = 4x^5 + 7x^7 - 16x^3$

b) $f(x) = 14 - 9x - 3x^2$

c) $f(x) = -2 + 5x^3 - 3x^5 + 16x^4$

23. What are the zeros of each function?

a) $f(x) = x^3 - 512$	b) $f(x) = x^3 - 3x^2 - 16x - 48$
c) $f(x) = 25x^3 + 25x^2 - 100x - 100$	d) $f(x) = x^4 - x^3 - 8x^2 - 4x - 48$
e) $f(x) = x^4 + 48x^2 - 49$	

24. Find a polynomial function that meets the following descriptions:

a) i) crosses the x-axis twice, once at 4, once at -3 ii) degree is 4 iii) no imaginary zeros $f(x) = \underline{\hspace{10em}}$	b) Polynomial of least degree that has zeros at $3, -1, i\sqrt{2}$
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25. Solve each inequality.

a) $x^3 - 4x^2 - 20x + 48 \geq 0$	b) $x^3 + 4x^2 < 7x + 10$
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26. Identify the asymptotes (Horizontal, Vertical, Slant/Oblique) of each function.

<p>a) $f(x) = \frac{3x+6}{x^2-16}$</p>	<p>b) $f(x) = \frac{x^2+5x+6}{x^2+6x+8}$</p>
<p>c) $f(x) = \frac{x^2+3x+11}{x+4}$</p>	<p>d) $f(x) = \frac{2x^2+7x+3}{x-3}$</p>

27. Describe what happens to the asymptotes of each function after the function undergoes a transformation.

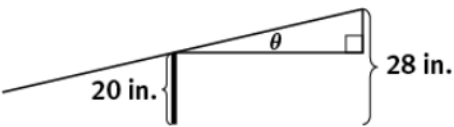
<p>Given: $f(x)$ has a vertical asymptote, $x = 3$, and a horizontal asymptote, $y = -6$. Transformation: $f(x - 3) + 3$</p>	<p>Given: $f(x)$ has a vertical asymptote, $x = -7$, and a horizontal asymptote, $y = 0$. Transformation: $f(x + 4) - 2$</p>
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UNIT 3

28. Find the exact value of each expression *without using a calculator*. Express your answer in radians.

<p>a) $\cos^{-1}\left(\frac{1}{2}\right)$</p>	<p>b) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$</p>	<p>c) $\tan^{-1}(-1)$</p>
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29. Solve each of the following:

<p>a) A seesaw is 6 feet in length. One side rises to a height of 28 inches. Find the measure of the angle of the elevation. If necessary, round to nearest tenth.</p> 	<p>b) Find an angle between 90° and 360° such that $\cos(x) = \cos 63^\circ$</p>
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a) $P(6, 8)$

b) $P(-2, -3)$

31. The angle θ specified is in standard position. O is the origin and the point P is on the terminal side of the angle. Given the length, what are the coordinates of the point P, to the nearest hundredth.

a) $\theta = 212^\circ$, $OP = 15$

b) $\theta = 321^\circ$, $OP = 12$

c) $\theta = 129^\circ$, $OP = 18$

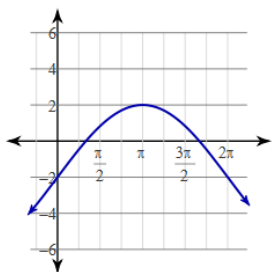
32. Solve each of the following

a) Find $\cot(x)$, given that $\cos(x) = \frac{2}{7}$ and the terminal side of the angle is in Quadrant 4.

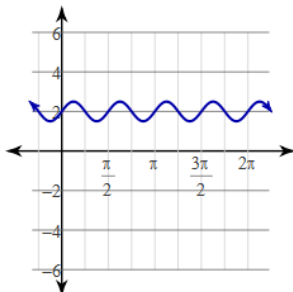
b) Given $\cos(x) = 0.524$, find $\sec(x)$, correct to three decimal places.

33. Match the following equation to its correct graph: $y = \frac{1}{2}\sin(4x + 2) - 2$.

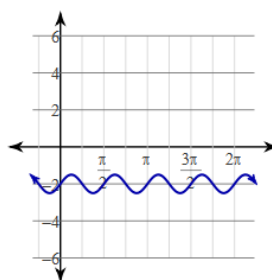
A)



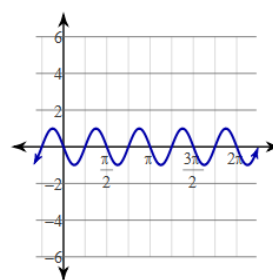
B)



C)



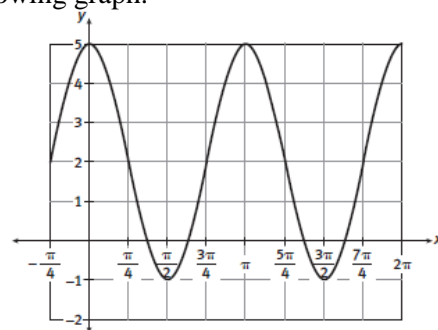
D)



34. Write two equations, one in terms of cosine and one in terms of sine, for the following graph.

sine: _____

cosine: _____



35. Solve each of the following on the interval $[0, 2\pi)$

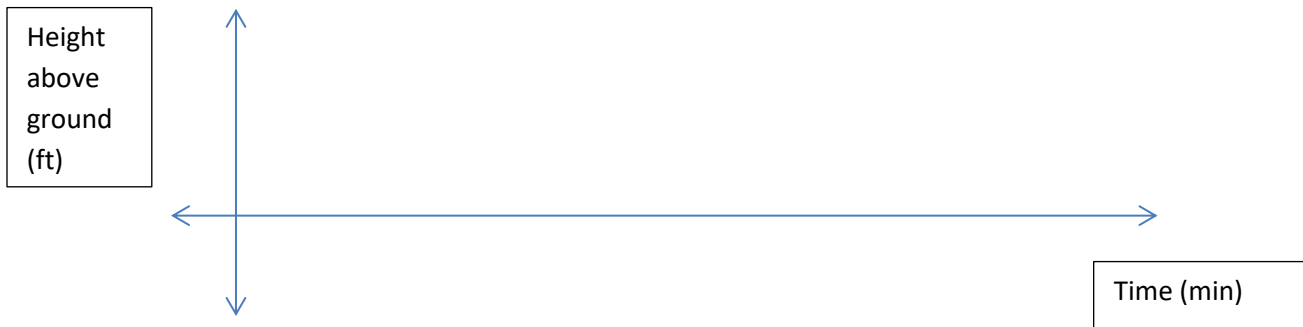
a) $\cos x - 2 \sin x \cos x = 0$	b) $3 \tan^3 x - \tan x = 0$
c) $\cos^2 x - 1 = 0$	d) $\tan x \sin x + \sin x = 0$
e) $\sin^2 3x - 1 = 0$	f) $2\cos^2 x + \cos x = 0$

36. Find the general solutions for each of the following:

a) $2\cos^2 x + \cos x = 0$	b) $4\sin^2 x - 3 = 0$	d) $\cos x \sin x - 2\cos x = 0$
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37. The “High Roller” Ferris Wheel is nearing completion in Las Vegas, NV. When it is done it will be the tallest Ferris Wheel in the world. It will reach a maximum height of 550 feet and will begin 30 feet off the ground. It is expected to take about 36 minutes to ride the entire attraction.

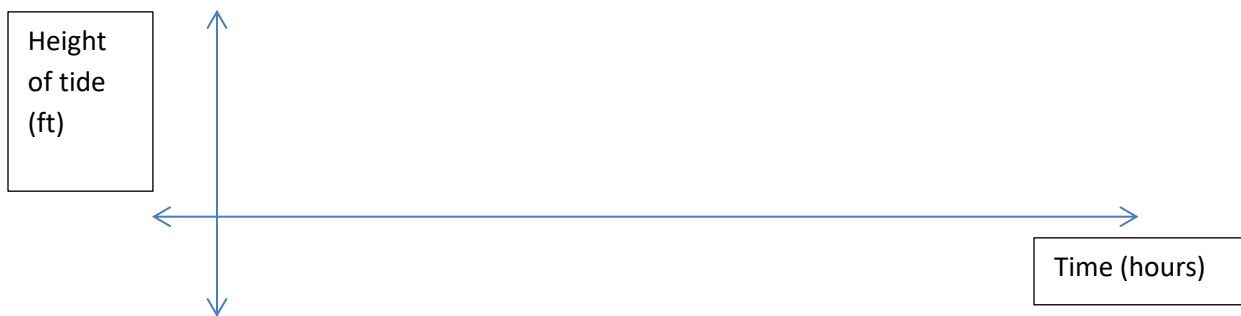
- a. Draw a graph to demonstrate the motion the Ferris wheel takes. Place time on the x-axis and distance above the ground on the y-axis.



- b. Write a cosine equation to describe your graph:

- c. Using your equation from part b, how high off the ground would you be 13 minutes into the ride?

38. Suppose the tidal range of a city on the Pacific coast reaches a maximum height at 12 feet and a low tide at about 1 foot. This pattern continues every 24 hours and models a cosine curve.



- a. Find a formula for the tidal range of this city as a function of time.

- b. How high will the tide be 30 hours into the cycle?

39. During a ride on the Navy Pier Ferris Wheel, the cycle is modeled by the function

$f(x) = -70 \cos\left(\frac{\pi}{5}x\right) + 80$, where x is measured in minutes into the ride. For approximately how many minutes are riders higher than 120 feet above the ground.

40. A certain person's blood pressure oscillates between 120 and 70. If the heart beats 30 times per minute, write a sine function that models the person's blood pressure.

41. In a certain region with hawks as predators and rodents as prey, the rodent population R varies according to the model $R = 20 \sin\left(\frac{\pi}{2}t\right) + 100$, and the hawk population H varies according to the model $H = 50 \sin\left(\frac{\pi}{2}t - \frac{\pi}{4}\right) + 300$, with t measures in years since January 1, 2000.

a) What are the maximum populations of rodents and hawks? Do these maximums ever occur at the same time?

b) What is the minimum population of hawks? On what date was the minimum population of hawks first achieved?

c) According to the models, what was the population of rodents and hawks on July 15th of the present year?

d) What is the amplitude of the hawk's model?

e) What is the midline of the rodent's model?

42. The tire of a tractor accidentally drives over a sticky substance. The substance sticks to the tire. The radius of the tire 40 inches and the wheel makes one revolution every 12 seconds. Find a cosine function that gives the height above the ground of the gum on the tire as a function of time (t) in hours. Then graph the function and determine at what time(s) the sticky substance is 30 inches off the ground.

43. Find all shifts, reflections, and amplitude changes for each equation.

<p>a) $f(x) = -6 \sin(4x + \frac{\pi}{2}) - 10$</p> <p>Amplitude: _____ Reflection? _____</p> <p>Period: _____</p> <p>Phase Shift: _____</p> <p>Vertical Shift: _____</p>	<p>b) $f(x) = 2 \tan(\pi x - \pi) + 2$</p> <p>Amplitude: _____ Reflection? _____</p> <p>Period: _____</p> <p>Phase Shift: _____</p> <p>Vertical Shift: _____</p>
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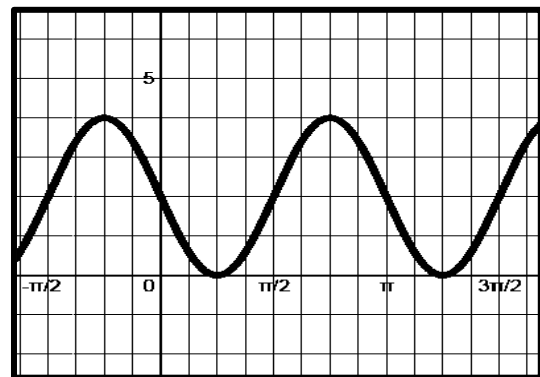
44. Write the equation of a cosine function given the following information:

<p>a) Amplitude = 7</p> <p>Phase Shift = π left (or $-\pi$)</p> <p>Vertical Shift = up 8</p> <p>Reflection = None</p> <p>Period = 2π</p> <p>$f(x) =$</p>	<p>b) Amplitude = 4</p> <p>Phase Shift = $\frac{3\pi}{4}$ right (or $\frac{3\pi}{4}$)</p> <p>Vertical Shift = up 8</p> <p>Reflection = over the midline</p> <p>Period = $\frac{\pi}{2}$</p> <p>$f(x) =$</p>
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45. Write a sine and a cosine equation of the following graph:

Sine:

Cosine:



46. State the period and amplitude and any shifts for each of the following trig functions and graph at least one period of the function.

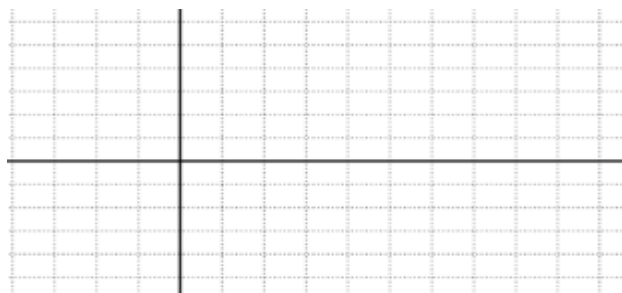
a) $y = -2\cos(4x)$

Period:

Vertical Shift:

Amplitude:

Horizontal Shift:



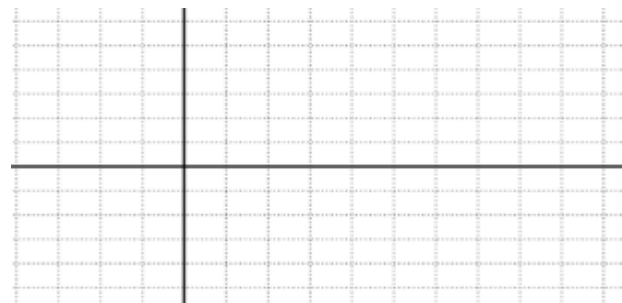
b) $y = 3 \csc 2x - 4$

Period:

Vertical Shift:

Amplitude:

Horizontal Shift:



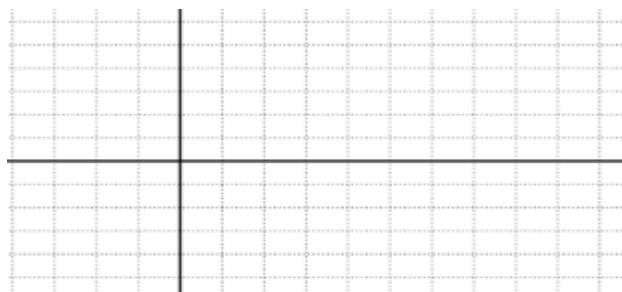
c) $y = -\tan\left(\frac{x}{3}\right)$

Period:

Vertical Shift:

Amplitude:

Horizontal Shift:



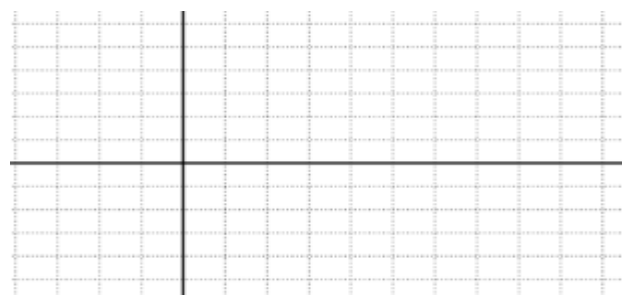
d) $y = 4 \sin\left(\frac{3}{4}x\right)$

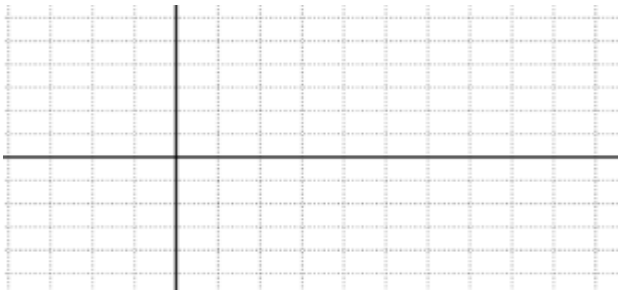
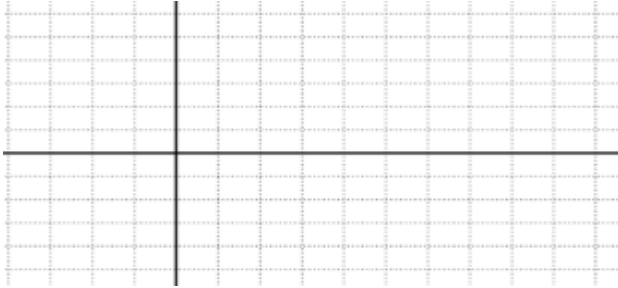
Period:

Vertical Shift:

Amplitude:

Horizontal Shift:



<p>e) $y = -\tan\left(2x - \frac{\pi}{2}\right)$</p> <p>Period: Vertical Shift:</p> <p>Amplitude: Horizontal Shift:</p> 	<p>f) $y = 3\sec(2x) - 2$</p> <p>Period: Vertical Shift:</p> <p>Amplitude: Horizontal Shift:</p> 
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47. Explain why $f(x) = \tan x$ is an odd function.

48. Which of the following trigonometric functions have no zeros? Explain.

$y = \sin x$	$y = \cos x$	$y = \tan x$
$y = \csc x$	$y = \sec x$	$y = \cot x$

49. Describe the transformation of $f(x) = \sec x$ to $g(x) = -\frac{1}{2}\sec(2x)$

50. What is the amplitude and period of $f(x) = 2 \cos\left(\frac{1}{2}x - 4\right) - 1$?

a. Amplitude: 1; Period: $\frac{1}{2}$

b. Amplitude: 2; Period: $\frac{1}{2}$

c. Amplitude: 2; Period: 4π

d. Amplitude: 1; Period: 4π

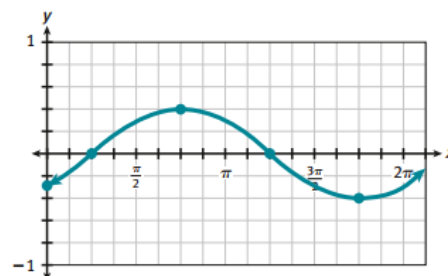
51. Which function could be represented by the graph shown?

a. $f(x) = \frac{1}{2}\sin\left(x - \frac{\pi}{4}\right)$

b. $f(x) = \frac{1}{2}\cos\left(x - \frac{\pi}{4}\right)$

c. $f(x) = \frac{1}{2}\sin\left(x + \frac{\pi}{4}\right)$

d. $f(x) = \frac{1}{2}\sin(x) - \frac{\pi}{4}$



52. Find one positive and one negative conterminal angle to 630°

53. Find the six trig ratios for an angle in standard position with terminal side at $(-2, 4)$

54. State all angles that make the following statement true: $\cos \theta = -\frac{\sqrt{2}}{2}$, $0 \leq \theta < 2\pi$.

55. There is a circle centered at the origin (O) with radius n . There is a point on the circle at $P(5.563, 2.248)$ which the terminal side of angle θ passes through. Given that the angle (θ) is 22° , find the radius of the circle.

56. A ramp projects to be in quadrant 1, with a vertical height of 3, and a ramp length of 10.

What is the angle of elevation, rounded to the nearest tenth.

57. A circular table saw takes 1 second to rotate and has a diameter of 6 inches.

If a specific saw tooth is bent and starts at the highest position, and the height is based on the height from the cutting surface, sketch a graph for the path of the bent tooth for 3 seconds.