Honors Pre-Calculus Unit 3 Review Name: Key

1. Convert the following degree measure to radians

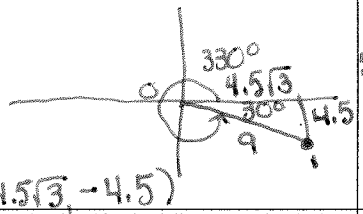
a. 50°	b. 190°	c. 275°
$50 \cdot \frac{\pi}{180} = \frac{5\pi}{18}$	$190 \cdot \frac{\pi}{180} = \frac{19\pi}{18}$	$275 \cdot \frac{\pi}{180} = \frac{55\pi}{36}$

2. Convert the following radian measure to degrees

a. $\frac{3\pi}{8} \cdot \frac{180}{\pi}$	b. $\frac{4\pi}{9} \cdot \frac{180}{\pi}$	c. $\frac{8\pi}{3} \cdot \frac{180}{\pi}$	d. $3 \cdot \frac{180}{\pi}$
67.5°	80°	480°	$\approx 171.9°$

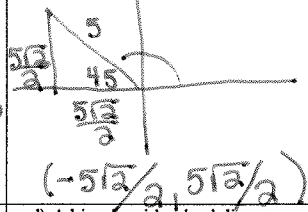
3. Solve each of the following:

a. If O is the origin of the coordinate plane, Let P be a point on the terminal side of an angle with a measure of 330°. If OP = 9, what are the coordinates of P?



$(4.5\sqrt{3}, -4.5)$

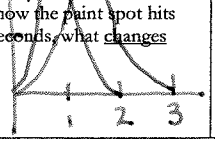
b. If O is the origin of the coordinate plane, Let Q be a point on the terminal side of an angle with a measure of 135°. If OP = 5, what are the coordinates of P?



$(-5\sqrt{2}/2, 5\sqrt{2}/2)$

c) A bicycle with wheel diameters of 20 inches runs over a line of paint. The paint spot touches the ground every 3 seconds. If the rider speeds up and now the paint spot hits the ground every 2 seconds, what changes occur in the graph?

period

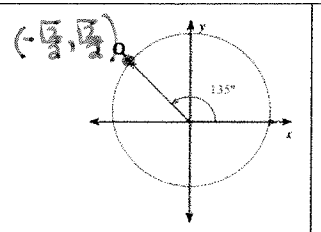


d) A bicycle with wheel diameters of 20 inches runs over a line of paint. If you compare this a graph of a bicycle that has wheel diameters of 17, what changes occur in the graph?

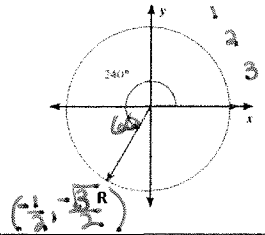
amplitude

4. The center of the Unit circle is located at the origin. Find the coordinates of the points.

a)



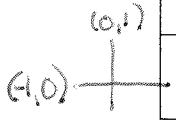
b)



5. The point P(a, b) is located on the unit circle in standard position on the terminal side of an angle θ. Write each of the six trig function in terms of a and b.

a) $\sin\theta = \frac{b}{\sqrt{a^2+b^2}}$	b) $\cos\theta = \frac{a}{\sqrt{a^2+b^2}}$	c) $\tan\theta = \frac{b}{a}$
d) $\csc\theta = \frac{\sqrt{a^2+b^2}}{b}$	e) $\sec\theta = \frac{\sqrt{a^2+b^2}}{a}$	f) $\cot\theta = \frac{a}{b}$

6. State the exact values of the following



a) $\sin\frac{11\pi}{6}$	b. $\cos\frac{7\pi}{6}$	c) $\tan\frac{3\pi}{4}$	d) $\csc\frac{2\pi}{3}$
$-1/2$	$-\frac{\sqrt{3}}{2}$	-1	$\frac{2\sqrt{3}}{3}$
e) $\tan\frac{\pi}{2}$	f) $\sec 5\pi$	g) $\cot\frac{\pi}{2}$	h. $\sin\frac{18\pi}{3}$
und.	$= \sec(5\pi - 4\pi)$ $= \sec\pi = -1$	0	$\sin(6\pi) = \sin 0$ $= 0$

$\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

7. State the exact values of the following:

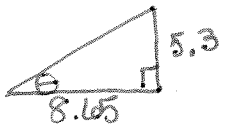
a) $\sin^{-1}\left(\frac{1}{2}\right)$ $\pi/6$	b) $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\pi/6$	c) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$
d) $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right)$ $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$	e) $\sin\left(\tan^{-1}(\sqrt{3})\right)$ $\sin(\pi/3) = \sqrt{3}/2$	f) $\tan^{-1}(\cos(\pi))$ $\tan^{-1}(-1) = -\pi/4$

8. List the angle that is between 0 *and* 2π that is co-terminal with the following angles

a) 585°	b) 1035°	c) $-300°$	d) 1200°
-360 $225°$	-720 $315°$	$60°$ $\pi/3$	$120°$ $\frac{2\pi}{3}$
$225° = 5\pi/4$	$7\pi/4$		

9) Solve each of the following:

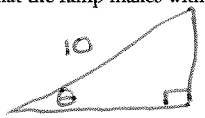
a)
A tree is 5.3 meters tall, with a shadow of 8.65 meters long. What is the angle of the sun's rays to the nearest tenth of a degree?



$\tan\theta = \frac{5.3}{8.65}$

$\theta = \tan^{-1}\left(\frac{5.3}{8.65}\right) \approx 31.5°$

b)
A skateboard ramp is 10 feet long and 4 feet high. What is the angle of elevation that the ramp makes with the ground?



$\sin\theta = \frac{4}{10}$

$\sin^{-1}(4/10) \approx 23.6°$

10. Determine whether each of the following situations could be represented with a periodic function.

a) The distance from point A to point B.	b) The cost of groceries over a length of time.	c) The average monthly temperature in Plainfield over a number of years.
no	no	yes

11. Solve the following trigonometric equations to the nearest tenth of a degree over the interval $[0°, 360°)$

a) $7\sin(\theta) - 4 = 1$

$7\sin\theta = 5$

$\sin\theta = 5/7$

$\theta \approx 45.6°$ or $134.4°$

b) $9\cos(\theta) + 2 = 5$

$9\cos\theta = 3$

$\cos\theta = 1/3$

$\theta \approx 70.5°$ or $289.5°$

12. Calculate the solutions of the following equations over the interval $[0, 2\pi)$. Give exact answers when possible. If rounding is necessary, round to the nearest thousandth.

a) $25\tan^2 x + 3 = 12$

$25\tan^2 x = 9$ $x = \tan^{-1}(3/5)$

$\tan^2 x = 9/25$

$\tan x = \pm 3/5$

$x \approx 0.540, 3.683, 3.683, 5.743$

b) $9\sin^2 x + 3 = 7$

$9\sin^2 x = 4$

$\sin^2 x = 4/9$

$\sin x = \pm 2/3$

$x = \sin^{-1}(2/3)$

$x \approx 0.730, 2.412, 3.872, 5.553$

13, Calculate the amplitude, period, phase shift, and vertical shift of the following equations

a) $f(x) = -2\cos(3x) - 1$

$f(x) = -2\cos(3x) - 1$

Amp: 2

Per: $2\pi/3$

P.S: none

V.S: down 1

b) $f(x) = 3\sin(\frac{1}{2}x) + 5$

$\frac{2\pi}{1/2} =$

Amp: 3

Per: 4π

P.S: none

V.S: up 5

c. $f(x) = \sin(2x + \frac{\pi}{4}) - 3$

$\sin 2(x + \pi/8) - 3$

Amp: 1

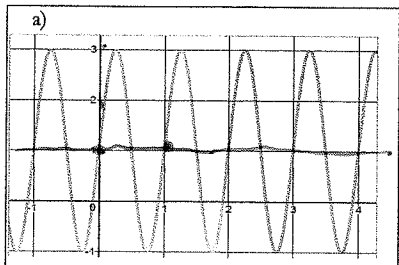
Per: π

P.S: left $\pi/8$

V.S: down 3

14) State the amplitude and period for each graph:

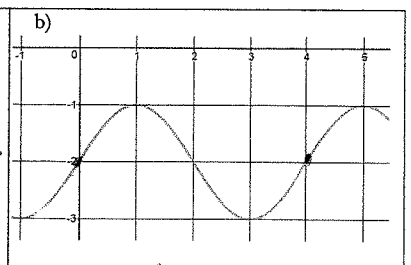
a)



Amplitude: 2

Period: 1

b)



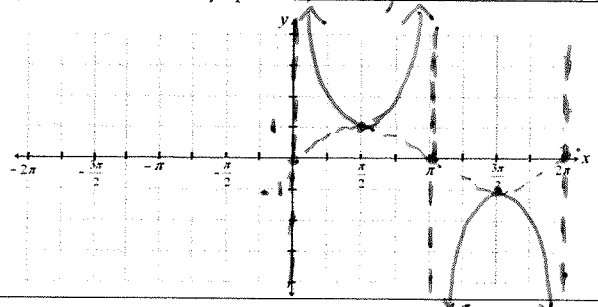
Amplitude: 1

Period: 4

Graph the function and write an equation for the vertical asymptotes

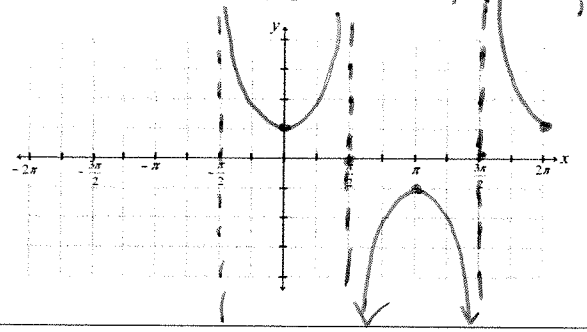
15

$f(x) = \csc(x)$ Asymptotes: $x = \pi n$, $n \in \mathbb{Z}$



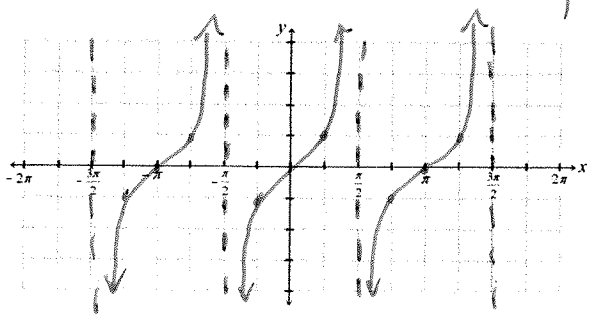
16.

$f(x) = \sec(x)$ Asymptotes: $x = \pi/2 + \pi n$; $n \in \mathbb{Z}$



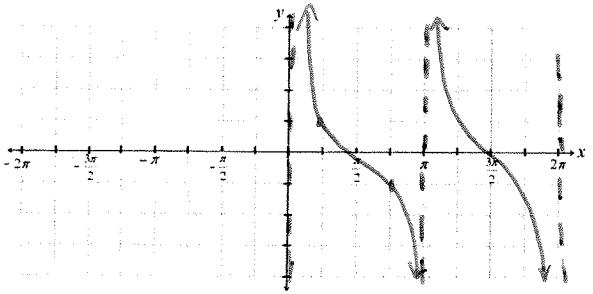
17.

$f(x) = \tan(x)$ Asymptotes: $x = \pi/2 + \pi n$, $n \in Z$



18.

$f(x) = \cot(x)$ Asymptotes: πn; $n \in Z$



19. The average daily low temperature in degrees Fahrenheit for a city is modeled by the function, where d is the number of days in the year. On which days is the average low temperature in the 40's?

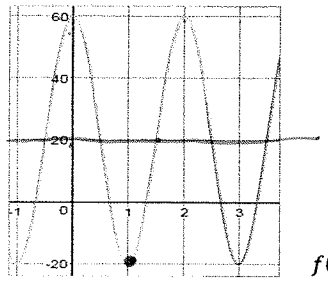
$$f(d) = 25.7\cos\left(\frac{2\pi}{365.25}d - 1\right) + 31.4$$

about 127 days and 351 days

put this equation in as y_1
and $y = 40$ as y_2
look at intersection

20. Write a Sine function for the given graph

42.

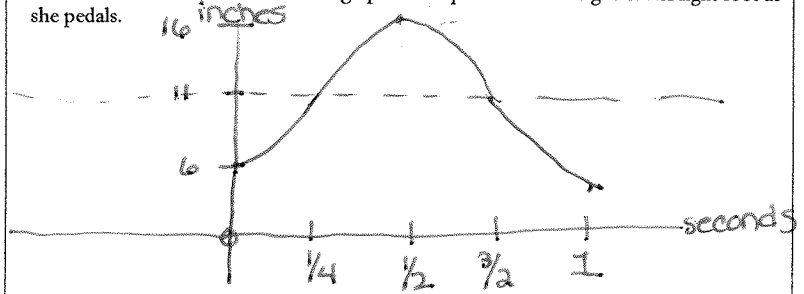


amp 40
shifted 1.5 right
up 20
Period 2 $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

$f(x) = 40\sin\pi(x - 1.5) + 20$
$= 40\sin(\pi x - 3\pi/2) + 20$
or $= -40\sin\pi(x - .5) + 20$

$-40\sin(\pi x - \pi/2) + 20$

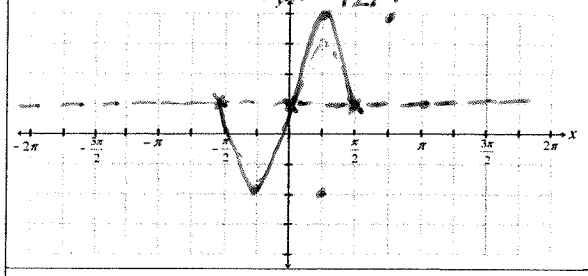
21. A cyclist rides a bike with pedals that are 6 inches to 16 inches off the ground. When the cyclist begins pedaling, her right foot is 6 inches off the ground and her feet make one revolution in 1 second. Draw a graph that represents that height of her right foot as she pedals.



Graph the following functions

22 $f(x) = -3\sin(2x + \pi) + 1$

$2(x + \pi/2) + 1$

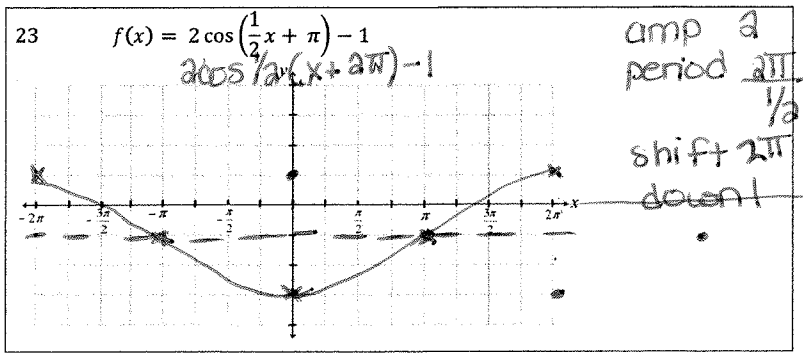


reflect
amp 3
period $\frac{2\pi}{2} = \pi$

$\pi/2$ left
up 1

23 $f(x) = 2\cos\left(\frac{1}{2}x + \pi\right) - 1$

$2\cos\frac{1}{2}(x + 2\pi) - 1$



amp 2

period $\frac{2\pi}{1/2} = 4\pi$

shift 2π Left

down 1

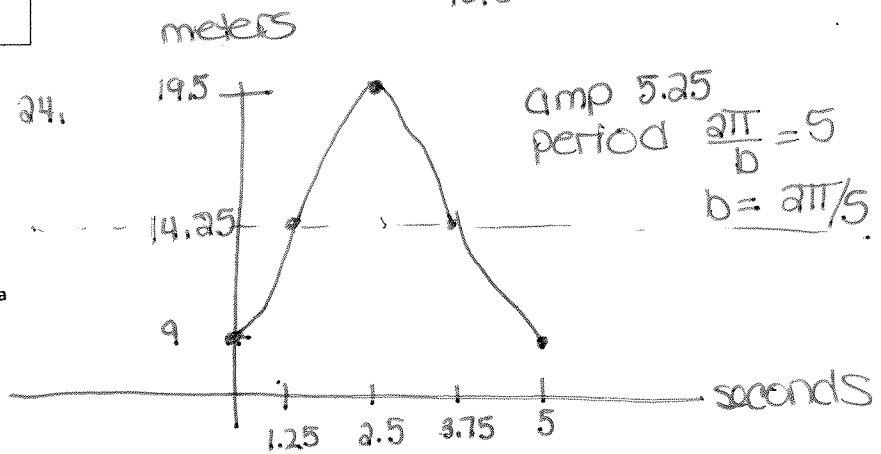
24. A boat bobs up and down on the ocean at a constant rate. It reaches a maximum height of 19.5 meters and a minimum height of 9 meters. The boat starts at its lowest position and returns to the same position every 5 seconds.

Write a cosine function to find the boats height in meters, h, at time t seconds.

$h(t) = 5.25\cos\frac{2\pi}{5}(x) + 14.25$

$19.5 - 9 = 10.5 \quad 5.25$

24.



amp 5.25

period $\frac{2\pi}{b} = 5$

$b = 2\pi/5$

25. The world's tallest Ferris wheel is the Singapore Flyer which measures 165 meters tall, and has a diameter of 150 meters. After you get on the Ferris wheel you reach the highest point after 23 minutes and then reaches its lowest point after 38 minutes.

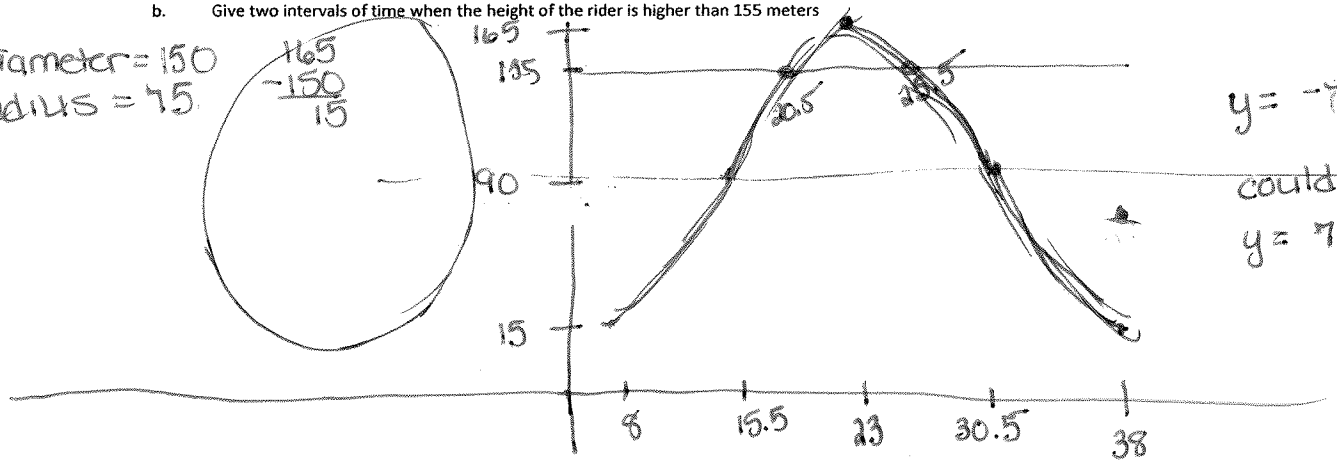
a. The height of a person on the Ferris wheel over time forms a sinusoidal curve. Find an equation for the height of a rider after getting on the wheel.

b. Give two intervals of time when the height of the rider is higher than 155 meters

diameter = 150

radius = 75

$165 - 150 = 15$



$y = -75\cos\pi/15(x - 8) + 90$

could also write

$y = 75\sin\pi/15(x - 15.5) + 90$

$\frac{2\pi}{b} = 30$

$30b = 2\pi$

$b = \pi/15$

$38 - 23 = 15$

$22 - 7.5 = 15.5 - 7.5 = 8$

b) (20.5, 25.5) and (50.5, 55.5)