Pre-Calculus Worksheet
Tangent and Cotangent

Name: ______________________

Per: ____

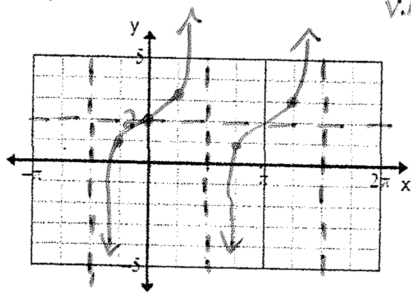
I. Fill in the chart for each function. DO NOT GRAPH. Remember to factor first when needed.

	1. $y=2\tan\left(\frac{1}{2}x\right)-3$	2. $y=\frac{4}{3}\cot\left(4\left(x-\frac{\pi}{2}\right)\right)+1$	3. $y=-5\tan(3x+\pi)$ $y=-5\tan 3(x+\pi/3)$
Amplitude:	none Rise/Fall: 2	none rise/Fall: 4/3	none rise/fall: 5
Flip?	no	no	yes
Vertical Shift:	down 3	up 1	none
Period:	$\frac{\pi}{1/2} = 2\pi$	$\frac{\pi}{4}$	$\pi/3$
Phase Shift:	none	Rt $\pi/2$	Left $\pi/3$

II. Graph each function, over one period, showing the vertical asymptotes.

4. $y=\tan x+2$

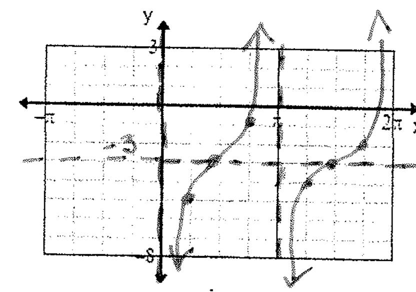
V.A. $x=\frac{\pi}{2}+n\pi, n\in\mathbb{Z}$



5. $y=2\cot x-3$

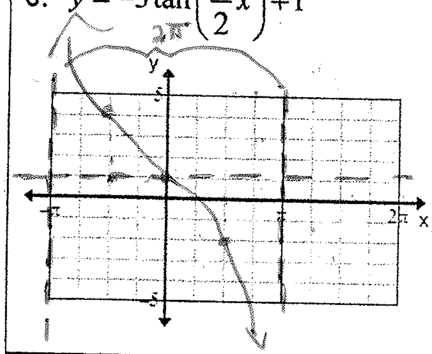
rise/fall: 2
down 3

V.A. $x=n\pi, n\in\mathbb{Z}$



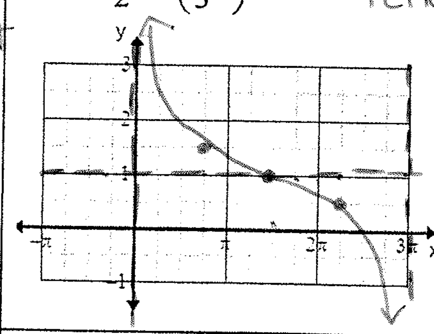
6. $y=-3\tan\left(\frac{1}{2}x\right)+1$

Reflected
Rise/fall 3
Period $\frac{\pi}{\frac{1}{2}}=2\pi$
up 1
VA: $x=\pi+n2\pi$; $n\in\mathbb{Z}$



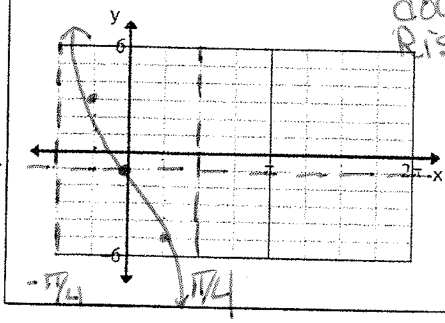
7. $y=\frac{1}{2}\cot\left(\frac{1}{3}x\right)+1$

Rise/fall 1/2
Period $\frac{\pi}{\frac{1}{3}}=3\pi$
Up 1
VA $x=3\pi n$; $n\in\mathbb{Z}$



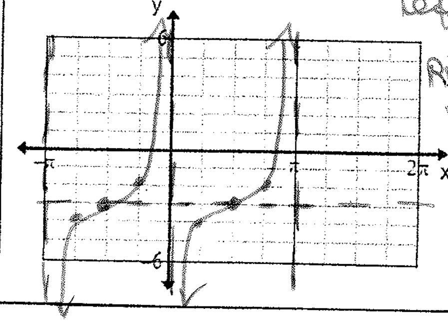
8. $y=-4\tan\left(x-\frac{\pi}{4}\right)-1$

Reflected
Rt $\pi/4$
down 1
Rise/Fall 4

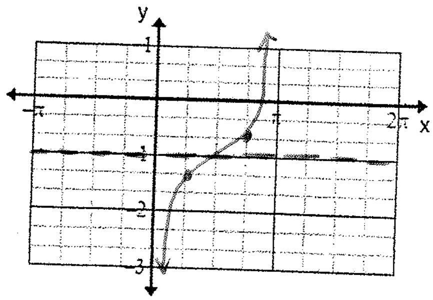


9. $y=-\cot(x+\pi)-3$

Reflected
period π
left π
down 3
Rise/fall: 1
VA: $x=n\pi$; $n\in\mathbb{Z}$



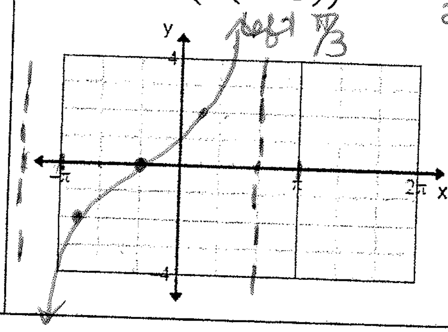
10. $y=-\frac{1}{3}\cot x-1$



11. $y=2\tan\left(\frac{1}{2}\left(x+\frac{\pi}{3}\right)\right)$

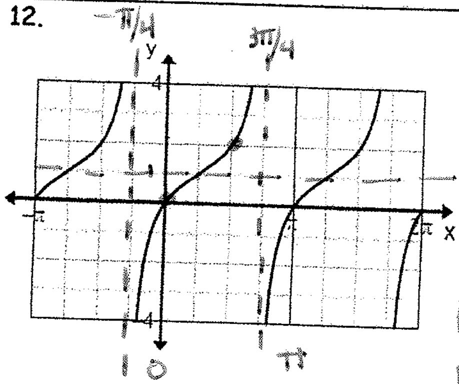
$\frac{\pi}{\frac{1}{2}}=2\pi$

left $\pi/3$



III. Write the equation for each trigonometric function.

12.



Period π

a = 1

tan

shifted

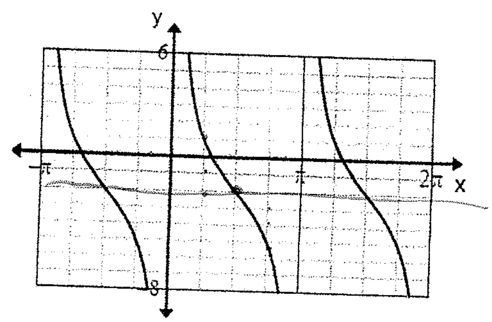
Rt π/4

Rise Fall 1

up 1

$f(x)=\tan\left(x-\frac{\pi}{4}\right)+1$

13.



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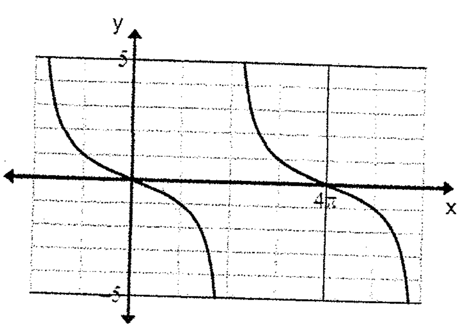

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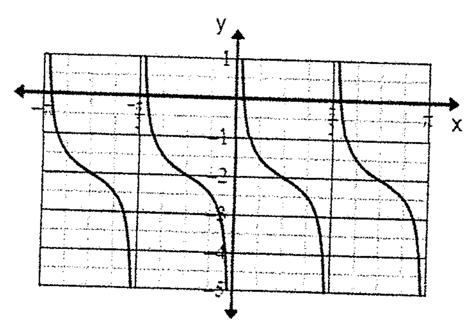

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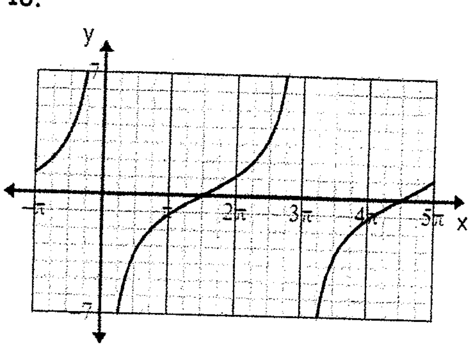

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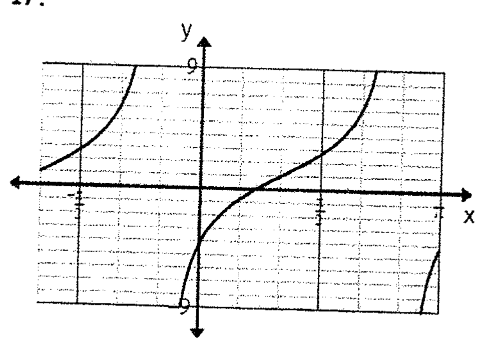


HONORS PRECALC
Writing Trig Equations

NAME________________________________
Period______________________

Specifying a sine (or cosine) curve with a given amplitude, period, and phase shift defines a unique set of points in the plane. However, there are infinitely many equations that can describe that set of points!
For example, the set of points given by the sine curve with amplitude 5, period 3, and phase shift 2 can be described by *any* of these equations (and many more):

$$y = 5\sin\frac{2\pi}{3}(x-2) \qquad y = -5\sin\frac{2\pi}{3}(x-3.5) \qquad y = 5\sin\frac{2\pi}{3}(x+1)$$

In this section, we find a simple (natural) equation that works, by using graphical transformations to change a basic model into a curve with the desired attributes.

The process is illustrated with an example:

EXAMPLE:

pos RT

$\frac{2\pi}{b} = 3$ $\frac{3b}{3} = \frac{2\pi}{3}$

Write an equation of a sine curve with amplitude 5, period 3, and phase shift 2.

SOLUTION:

- **Start with the basic model (sine or cosine):**
  We want a sine curve, so the 'basic model' is: $y = \sin x$
- **Apply a vertical stretch/shrink to get the desired amplitude:**
  new equation: $y = 5\sin x$
- **Apply a horizontal stretch/shrink to get the desired period:**
  - For $k > 0$, the curve $y = \sin kx$ has period $\frac{2\pi}{k}$.
  - We want the period to be 3: $\frac{2\pi}{k} = 3 \implies k = \frac{2\pi}{3}$
  - new equation: $y = 5\sin\frac{2\pi}{3}x$
- **Apply the desired phase shift:**
  To shift right 2, replace every x by $x-2$. Remember—transformations involving x are counter-intuitive!
  new equation: $y = 5\sin\frac{2\pi}{3}(x-2)$
- Thus, the curve $y = 5\sin\frac{2\pi}{3}(x-2)$ has amplitude 5, period 3, and phase shift 2.

$y = \underline{a}\sin \underline{b}(x-\underline{c}) + \underline{d}$
$5\sin\frac{2\pi}{3}(x-2)$
$-5\sin\frac{2\pi}{3}(x-2)$
$= \pm 5\sin\frac{2\pi}{3}(x-2)$
$= \pm 5\sin\left(\frac{2\pi}{3}x - \frac{4\pi}{3}\right)$

Important: Change the period BEFORE the phase shift!

If you apply the phase shift first, then the subsequent horizontal stretch/shrink to adjust the period will mess up this phase shift.
So, be sure to adjust the period *before* applying the phase shift!
The amplitude adjustment (the vertical stretch/shrink transformation) can be applied at any time.

Here is a second example:

EXAMPLE:

Write an equation of a cosine curve with amplitude 4, period $\frac{\pi}{3}$, and phase shift $-\frac{1}{2}$.

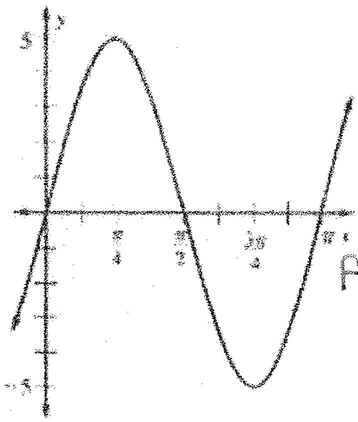
SOLUTION:

- basic model: $y = \cos x$
- period: $\frac{2\pi}{k} = \frac{\pi}{3} \implies k = 6$
  new equation: $y = \cos 6x$
- phase shift: $y = \cos 6(x + \frac{1}{2})$
- amplitude: $y = 4\cos 6(x + \frac{1}{2})$
- Thus, the curve $y = 4\cos 6(x+\frac{1}{2})$ has amplitude 4, period $\frac{\pi}{3}$, and phase shift $-\frac{1}{2}$.

PROBLEMS:

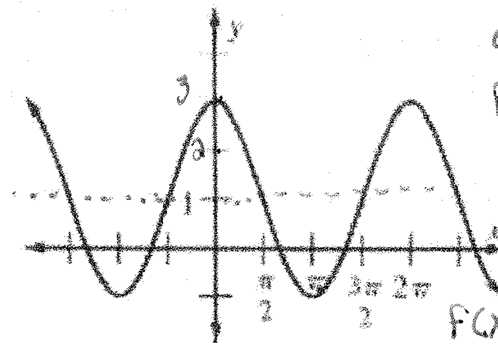
Examine the graph below and determine the amplitude, period, phase shift, and vertical shift of each using COSINE as the parent function. Then write an equation of the function.

1.



amp: 5
period: π $\frac{2\pi}{b} = \pi$ 2.
P.S. $\pi/4$ rt $b=2$
V.S. none

$f(x) = 5\cos 2(x - \pi/4)$

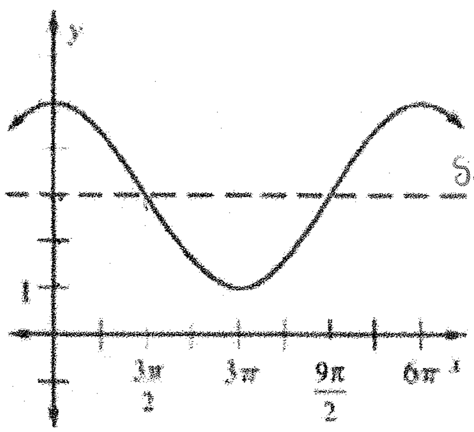


amp 2
period 2π
P.S. none
V.S. 1

$f(x) = 2\cos(x) + 1$

Examine the graph below and determine the amplitude, period, phase shift, and vertical shift of each using SINE as the parent function. Then write an equation of the function.

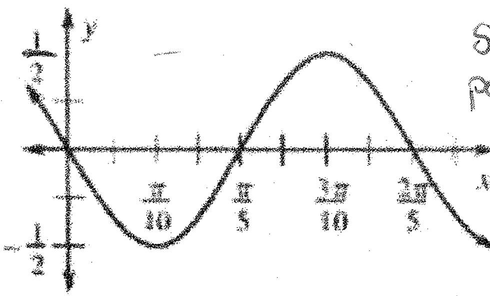
3.



sin Reflected:
amp 1
period: 6π
$\frac{2\pi}{b} = 6\pi$
$b = 1/3$
$3\pi/2$ RT
V.S 3

$f(x) = \sin 1/3(x - 3\pi/2) + 3$

4.



sin Ref
period: $\frac{2\pi}{5}$
$\frac{2\pi}{b} = \frac{2\pi}{5}$
$b = 5$
amp 1/2
V.S. none
P.S. none

$f(x) = -\frac{1}{2}\sin\frac{2\pi}{5}(x)$

Given the following information about each trig function, write a possible equation for each.

5.

$f(x) = a\sin b(x - c) + d$

Sine Function

$f(x) = \frac{1}{2}\sin 6(x) - 4$

amplitude $= \frac{1}{2} = a$

period $= \frac{\pi}{3}$ $\frac{2\pi}{b} = \pi/3$ $\pi b = 6\pi$
$b = 6$

vertical shift $= -4$

$c = 0$

$d = -4$

6.

$f(x) = a\cos b(x - c) + d$

Cosine Function
amplitude $= 3$
period $= \pi$ $\frac{2\pi}{b} = \pi$ $b = 2$
phase shift $= -\pi$ c
vertical shift $= -1.5$ d

$f(x) = 3\cos 2(x + \pi) - 1.5$