Honors Precalculus
UNIT 2 Review

NAME Key
PERIOD 8

This assessment covers many concepts which you must be able to understand without the use of your calculator to view the graph. Please complete the following table with explanations of how you are able to determine each concept without the use of your calculator:

Concept	**How to do with no calculator**
1) When looking at a data set without the calculator what are the key elements to consider if the data represents a: a) Quadratic b) Cubic c) Quartic	1a) at most 1 turning point maximum 2 zeros end behavior → or 1b) At most 2 turning pts Maximum of 3 zeros End behavior → or 1c) At most 3 turning pts maximum of 4 zeros End behavior → or
2) Given a data set, how might you determine where a new piece of data might fall: a) Using x- values b) Using y-values	2a) Given the x-value look at what x-values it falls between in the table. The y-value will be between the y-values of those x's and closer to the one the x-value is closest to 2b) Given the y-value look at which y-value it falls between and the x will fall between the x's of those values and closer to the one the y is closer to
3) For a POLYNOMIAL in STANDARD FORM: Without using a calculator, how can you determine: a) Domain b) Range c) End-behavior of a polynomial d) x-intercept(s) e) y- intercept f) Degree of polynomial g) Maximum number of zeros h) Multiplicity of zeros i) Maximum number of turning points j) Possible rational zeros k) Zeros (given multiple choice answers and after finding possible rational zeros	STANDARD FORM: when its term of highest degree is first, 2nd highest is 2nd, etc. 3a) $(-\infty, \infty)$ 3b) Degree is odd: $(-\infty, \infty)$ Degree is even $(\min, \infty)$ OR $(-\infty, \max)$ 3c) even deg pos l.c. even deg neg l.c. odd deg pos l.c odd deg neg l.c. 3d) Let $y=0$ and solve for x 3e) Let $x=0$ and solve for y 3f) highest exponent 3g) same as degree 3h) the number of times a factor occurs (more than once) 3i) one less than the degree 3j) $\frac{\text{factors of } p \text{ (constant)}}{\text{factors of } q \text{ (l.c.)}}$ then all possible $\frac{p}{q}$ 3k) Use synthetic division with choices given in m.c. answers until you get it to a quadratic

<table>
<tr>
<td>l) Find the y-value of another point (x)
m) The difference in the meaning of the directions when finding
i) rational zeros,
ii) real zeros
iii) all zeros
n) Factors of a polynomial
o) Relative maximum / Relative minimum (given a graph)
p) Solve a polynomial inequality</td>
<td>3l) use synthetic division with that x-value. The remainder is the y-value.
3mi) only rational zeros where $\frac{p}{q}$ are integers $q \neq 0$
3mii) only real zeros (no imaginary)
3miii) all complex zeros (both real and imaginary)
n) If a is a zero (x-a) is a factor
o) 
if function goes inc to dec → rel max
dec to inc → rel min
p) Find the real zeros (critical values) and test values in all intervals or use the number line method</td>
</tr>
<tr>
<td>4) For a RATIONAL FUNCTION:
Without using a calculator, how can you determine:

a) Domain
b) Range
c) End-behavior
d) x-intercept(s)
e) y- intercept
f) ~~x-asymptote~~ vertical asymptote
g) ~~y-asymptote~~ horizontal asymptote
h) slant asymptote
i) Find the y-value of another point (x)
j) Solve an inequality
K) hole. (removable discontinuity)</td>
<td>4a) All reals except any value that makes denominator equal zero
4b) consider graph (sketch)
4c) consider graph (sketch) may need to plug in some values
4d) Let y=0 and solve for x. (any value that makes only the numerator equal zero)
4d) Let x=0 and solve for y
4f) Any value that makes only the denominator equal zero
4g) compare deg of num to deg of den.
num bigger: none den bigger: y=0
same: $\frac{\text{l.c. of num}}{\text{l.c. of den}}$
4h) deg of num exactly 1 more than deg of den. do long division to find.
4i) Plug in the x-value and solve for y
4j) use "line method" marking all critical values. Exclude values that make den=0
K) Any value that makes both the numerator and denominator equal zero.</td>
</tr>
</table>

Honors Pre-Calculus Unit 2 Review

Name: ______________________________

Match each table with a function that best models the data.

C 1. $f(x) = -x^3 + 3x + 2$

D 2. $f(x) = x^3 + 3x - 4$

A 3. $f(x) = \frac{1}{4}x^2 + x - 4$

B 4. $f(x) = x^2 + x + 2$

A.

X	Y
-6	-1
-4	-4
-2	-5
0	-4
2	-1
4	4

quadratic y-int -4

B.

X	Y
-4	14
-2	4
0	2
2	8
4	22
6	44

Quad y-int 2

C.

X	Y
-6	200
-4	54
-2	4
0	2
2	0
4	-50

cubic y int 2
neg LC

D.

X	Y
-4	-80
-2	-18
0	-4
2	10
4	72
6	230

5. Examine the data in the table. What type of function could be used to model the data? Explain your reasoning.

University of XYZ										
Year	2001	2002	2004	2006	2010	2011	2012	2014	2015	2016
Number of Enrolled Students (in thousands)	10	10.5	12	15.6	20.1	21.2	22.7	25	30.5	32.8

cubic

Use the below table to answer questions 5 and 6. The table shows values from the function $J(s) = .004s^3$ which demonstrates the relationship of water speeds, s, required to create 1 joule of energy.

Speed	0	20	40	60	80	100	120	140
Joules	0	32	256	864	2048	4000	6912	10976

6. Approximately how much water speed is needed to generate 2500 joules of energy?

around 85

7. Approximately how much water speed is needed to generate 100 joules of energy?

around 30

Use the table below to answer questions 7 and 8. The table shows the average speed y (in feet per second) of a space shuttle for different times t (in seconds) after launch.

t	10	20	30	40	50	60	70	80
y	202.4	463.4	748.2	979.3	1186.3	1421.3	1795.4	2283.5

8. Approximately how many seconds would pass when the average speed would be 800 ft/sec? between 30 and 40, closer to 30
approximately 32 or 33 sec

9. Approximately what would the average speed be after 55 seconds?

approximately 1300 ft/sec

Match the function to the graph

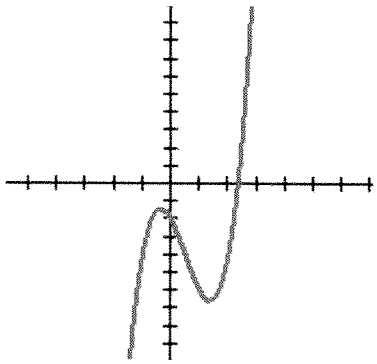
B 10. $f(x) = -2x^3 + 3x^2 + 3x - 2$

D 11. $f(x) = 2x^3 - 3x^2 - 3x + 2$

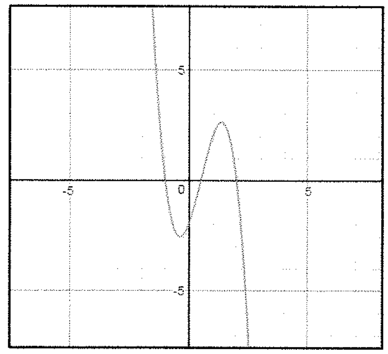
A 12. $f(x) = 2x^3 - 3x^2 - 3x - 2$

C 13. $f(x) = -x^3 - 2x^2 + x + 2$

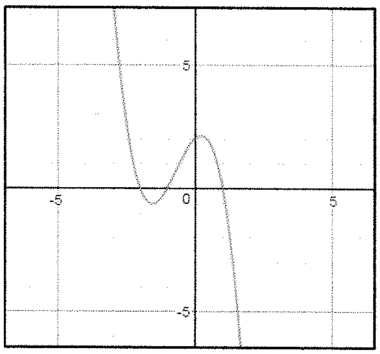
A.



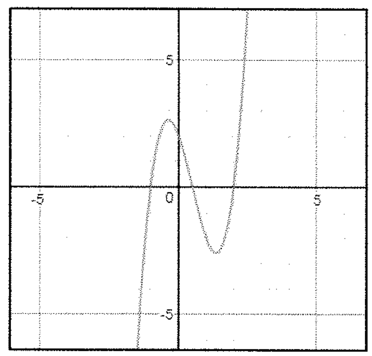
B.



C.



D.



Match the function to the graph

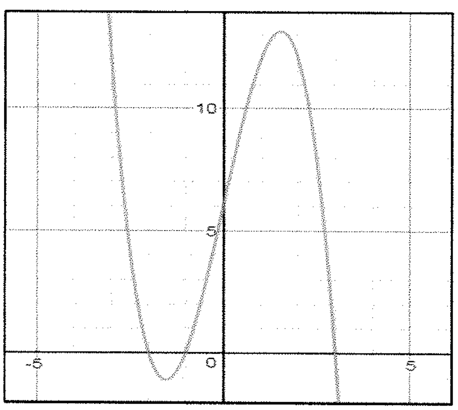
C 14. . $f(x) = x^3 - 7x + 6$

A 15. $f(x) = -x^3 + 7x + 6$

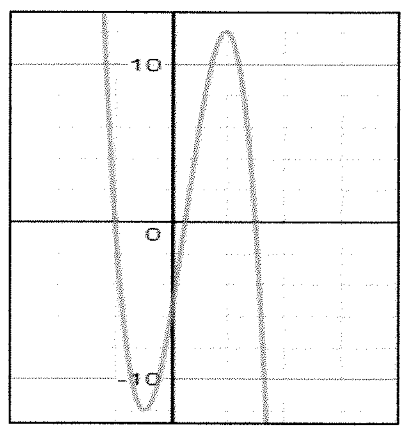
D 16. $f(x) = 2x^3 - 3x^2 - 11x + 6$

B 17. $f(x) = -2x^3 + 3x^2 + 11x - 6$

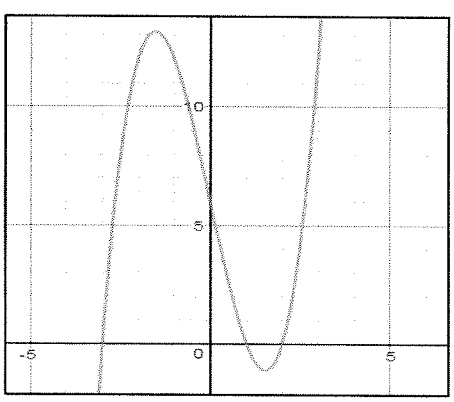
A.



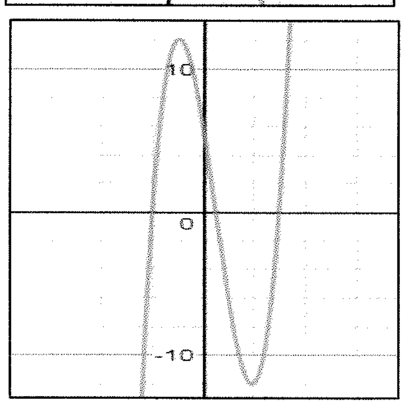
B.



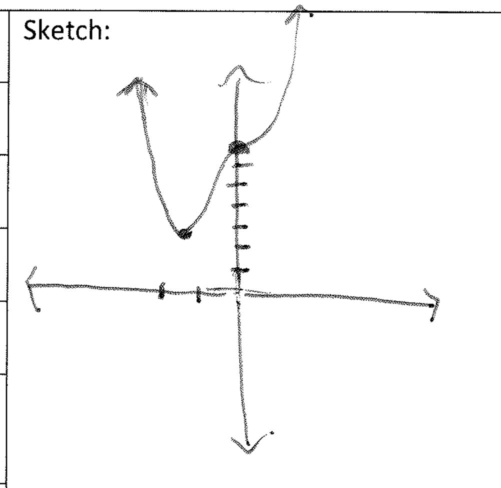
C.



D.



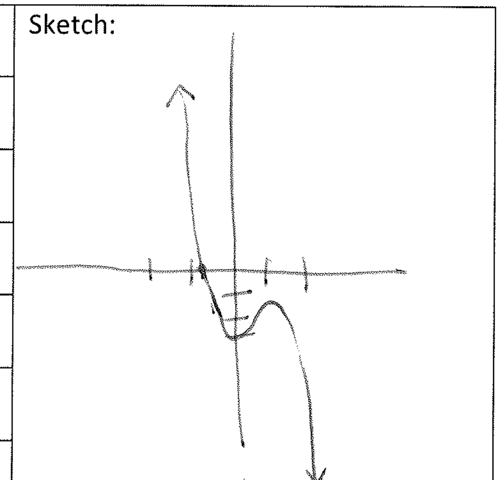
18. Given the equation $f(x) = 3x^3 + 2x^4 + 7 - 2x^2$, list the features below:

Standard Form: $f(x) =$ $2x^4 + 3x^3 - 2x^2 + 7$	Sketch: 
y-intercept: $(0, 7)$	
Maximum # of zeroes: 3	
Maximum # of turning points: 2	
Domain: $(-\infty, \infty)$	
Range: (NOTE: you will be given a graph for this if the degree is even) $(2.487, \infty)$	
End Behavior: ~~$f(x) \to \infty, x \to$ ∞ and $f(x) \to -\infty, x \to$ $-\infty$~~	(note: needed calc for this sketch)

as $x \to -\infty$, $f(x) \to \infty$ as $x \to \infty$, $f(x) \to \infty$

19. Given the equation $f(x) = x^3 - 5x^5 - 3 + 5x^2$, list the features below:

$f(-x) = 5x^4 + x^3 + 5x^2 - 3$

Standard Form: $f(x) =$ $-5x^5 + x^3 + 5x^2 - 3$	Sketch: 
y-intercept: $(0, -3)$	
Maximum # of zeroes: 5	
Maximum # of turning points: 4	
Domain: $(-\infty, \infty)$	
Range: $(-\infty, \infty)$	
End Behavior: ~~$f(x) \to \infty, x \to$ ____ and $f(x) \to -\infty, x \to$ ____~~	(note: used calc for this sketch)

as $x \to -\infty$, $f(x) \to \infty$; as $x \to \infty$, $f(x) \to -\infty$

20. Without using a calculator, determine the end behavior and x- and y- intercepts of the function

$$f(x) = (2x - 1)(x + 1)(x + 3)$$

$2x^3$

as $x \to -\infty$, $f(x) \to -\infty$ as $x \to \infty$, $f(x) \to \infty$

x-int: $(\frac{1}{2}, 0)$, $(-1, 0)$, $(-3, 0)$ y-int: $(0, -3)$

21. Without using a calculator, find the end behavior, maximum possible zeros, and maximum possible turning points of the function $f(x) = x^6 - 2x^3 + 1$

as $x \to -\infty$, $f(x) \to \infty$ as $x \to \infty$, $f(x) \to \infty$

max # of possible zeros: 6

max # of turning points: 5

Match the function to the graph

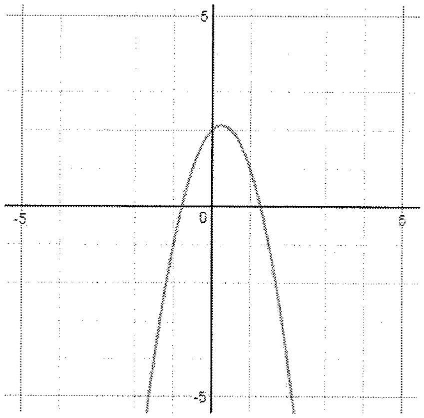
B 22. $f(x) = -4x^4 + x^2 - 2$

A 23. $f(x) = -2x^2 + x + 2$

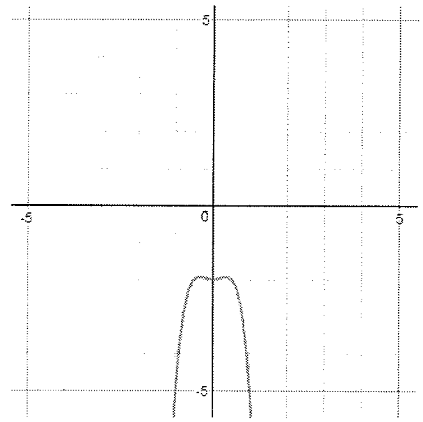
D 24. $f(x) = x^4 + x^2 + 2$

C 25. $f(x) = 2x^2 + x - 2$

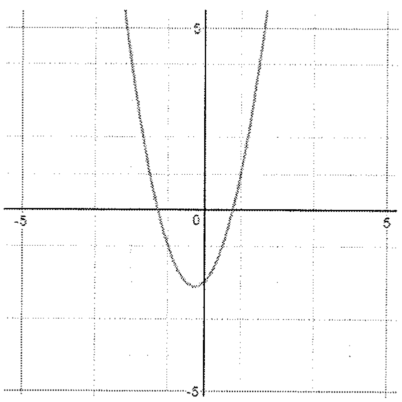
A.



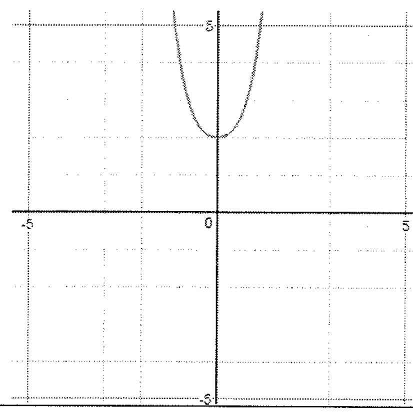
B.



C.



D.



26. Factor and find the zeros of the function: $g(m) = m^4 - 2m^3 + 8m - 16$

zeros: $\pm 2, 1 \pm i\sqrt{3}$

$m^3(m-2)+8(m-2)$

$(m^3+8)(m-2)$

$= (m+2)(m^2-2m+4)(m-2) = (m+2)(m-2)(m-(1+i\sqrt{3}))(m-(1-i\sqrt{3}))$

$m = \frac{2 \pm \sqrt{4-4(1)(4)}}{2} = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2}$

$= 1 \pm i\sqrt{3}$

27. Which of the following are possible zeros of $f(x) = 3x^4 + x^2 - 8$?

a) $\pm\frac{8}{3}$ b) $\pm\frac{3}{2}$ c) $\pm\frac{1}{2}$ d) $\pm\frac{16}{3}$

p: ± 1 ± 2 ± 4 ± 8

q: ± 1 ± 3

$\frac{p}{q}$: ± 1 $\pm\frac{1}{3}$ ± 2 $\pm\frac{2}{3}$ ± 4 $\pm\frac{4}{3}$ ± 8 $\pm\frac{8}{3}$

28. What are the zeroes of the function:
$f(x) = x^4 + 4x^3 + 8x^2 + 20x + 15$

A) $-3, 1, i\sqrt{5}, -i\sqrt{5}$

B) $-3, -1, i\sqrt{5}, -i\sqrt{5}$

C) $-3, -1, 5i, -5i$

D) $-3, 1, 5i, -5i$

$-3 \,|\, 1 \;\; 4 \;\; 8 \;\; 20 \;\; 15$; $\;\; -3 \;\; -3 \;\; -15 \;\; -15$

$-1 \,|\, 1 \;\; 1 \;\; 5 \;\; 5$; $\;\; -1 \;\; 0 \;\; -5$; $\;\; 1 \;\; 0 \;\; 5$

$x^2 = -5$

$x = \pm i\sqrt{5}$

29. What are the zeroes of the function:
~~$f(x) = x^4 + 4x^3 + 8x^2 + 20x + 15$~~
$f(x) = x^4 - x^3 + 23x^2 - 25x - 50$

A) $-1, -3, i\sqrt{5}, -i\sqrt{5}$

B) $-1, 2, i\sqrt{5}, -i\sqrt{5}$

C) $-1, -3, 5i, -5i$

D) $-1, 2, 5i, -5i$

$-1 \,|\, 1 \;\; -1 \;\; 23 \;\; -25 \;\; -50$; $\;\; -1 \;\; 2 \;\; -25 \;\; 50$

$2 \,|\, 1 \;\; -2 \;\; 25 \;\; -50 \;\; 0$; $\;\; 2 \;\; 0 \;\; 50$; $\;\; 1 \;\; 0 \;\; 25 \;\; 0$

$x^2 + 25 = 0$

$x^2 = -25$ $\quad x = \pm 5i$

30. List all of the possible real zeroes of the function
$f(x) = 2x^4 + x^3 + 2x + 8$

p: 8 $\quad \pm 1 \;\; \pm 2 \;\; \pm 4 \;\; \pm 8$

q: 2 $\quad \pm 1 \;\; \pm 2$

$\frac{p}{q}$ $\quad \pm 1, \pm \frac{1}{2}, \pm 2, \pm 4, \pm 8$

31. List all of the possible real zeroes of the function
$f(x) = 3x^3 + 4x^2 + 5x + 6$

p 6 $\quad \pm 1 \;\; \pm 2 \;\; \pm 3 \;\; \pm 6$

q 3 $\quad \pm 1 \;\; \pm 3$

$\frac{p}{q}$ $\quad \pm 1, \pm \frac{1}{3}, \pm 2, \pm \frac{2}{3}, \pm 3, \pm 6$

32. Using synthetic division, find all the zeros of the function below given that -1 is a zero with multiplicity 2.

{-1 and 3/2}
Both with mult. 2

$g(x) = 4x^4 - 4x^3 - 11x^2 + 6x + 9$

$-1 \,|\, 4 \;\; -4 \;\; -11 \;\; 6 \;\; 9$; $\;\; -4 \;\; 8 \;\; 3 \;\; -9$

$-1 \,|\, 4 \;\; -8 \;\; -3 \;\; 9 \;\; 0$; $\;\; -4 \;\; 12 \;\; -9$; $\;\; 4 \;\; -12 \;\; 9 \;\; 0$

$4x^2 - 12x + 9 = 0$

$(2x-3)(2x-3)$

$x = 3/2$

33. Determine the number of possible positive, negative and imaginary zeros for the functions below.

a) $f(x) = 5x^4 + 11x^3 + 6$ $\quad$ 0 pos

$f(-x) = 5x^4 - 11x^3 + 6$ $\quad$ 2 or 0 neg

b) $g(x) = 2x^5 - 4x^4 + x - 10$ $\quad$ 3 or 1 pos

$g(-x) = -2x^5 - 4x^4 - x - 10$ $\quad$ one neg

a)

pos	neg	imag
0	0	4
0	2	2

b)

pos	neg	imag
3	0	2
1	0	4

34. What is the complete factorization of the polynomial: $x^3 + 3x^2 - 6x - 8$

A) $(x-2)\ (x-4)\ (x+1)$

B) $(x+2)\ (x+4)\ (x+1)$

C) $(x-2)\ (x+4)\ (x+1)$

D) $(x-2)\ (x-4)\ (x-1)$

$2 \,|\, 1 \;\; 3 \;\; -6 \;\; -8$; $\;\; 2 \;\; 10 \;\; 8$; $\;\; 1 \;\; 5 \;\; 4$

$(x-2)(x+1)(x+4)$

35. What is the complete factorization of the polynomial, $x^3 - 2x^2 - 13x - 10$

A) $(x-1)\ (x+2)\ (x+5)$

B) $(x-5)\ (x + 1)\ (x+2)$

C) $(x-5)\ (x-2)\ (x+1)$

D) $(x-5)\ (x+1)\ (x+2)$

$5 \,|\, 1 \;\; -2 \;\; -13 \;\; -10$; $\;\; 5 \;\; 15 \;\; 10$; $\;\; 1 \;\; 3 \;\; 2$

$(x-5)(x+2)(x+1)$

36. Form a polynomial function of least degree with the following zeros: 5i, -2 (multiplicity 2)

$= (x+2)(x+2)(x-5i)(x+5i)$

$= (x^2+4x+4)(x^2+25)$

$= x^4 + 4x^3 + 4x^2$
$\quad + 25x^2 + 100x + 100$

$f(x) = x^4 + 4x^3 + 29x^2 + 100x + 100$

37. Form a polynomial function of least degree with the following zeros: -3, 4, 2 + 3i

$(x+3)(x-4)(x-(2+3i))(x-(2-3i))$

$(x^2 - x - 12)(x^2 - 4x + 13)$

$x^4 - x^3 - 12x^2$
$-4x^3 + 4x^2 + 48x$
$+13x^2 - 13x - 156$

$13 + 12 = 26$; 13

$f(x) = x^4 - 5x^3 + 5x^2 + 35x - 156$

38. Which intervals are part of the solution to the following inequality? **Select all that apply.**

$$2x^3 - 3x^2 - 11x + 6 \geq 0$$

A) $x \leq -2$

B) $-2 \leq x \leq \frac{1}{2}$

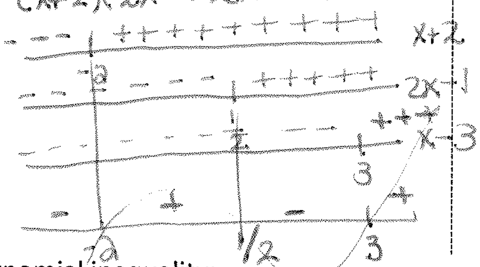
C) $x \leq \frac{1}{2}$

D) $\frac{1}{2} \leq x \leq 3$

E) $x \geq 3$

$-2 \,|\, 2 \;\; -3 \;\; -11 \;\; 6$
$\;\;\;\;\;\; -4 \;\; 14 \;\; -6$
$\;\;\;\; 2 \;\; -7 \;\; 3$

$(x+2)(2x-1)(x-3)$



$[-2, \frac{1}{2}] \cup [3, \infty)$

39. Which intervals are part of the solution to the following inequality? **Select all that apply**

$$\cancel{2x^3 - 3x^2 - 11x + 6 \leq 0}$$

$2x^3 - 3x^2 - 14x + 15 \leq 0$

A) $x \leq -\frac{5}{2}$

B) $-\frac{5}{2} \leq x \leq 1$

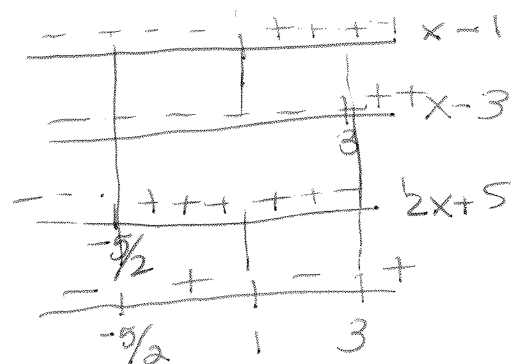
C) $x \leq 1$

D) $1 \leq x \leq 3$

E) $x \geq 3$

$1 \,|\, 2 \;\; -3 \;\; -14 \;\; 15$
$\;\;\;\;\;\; 2 \;\; -1 \;\; -15$
$\;\;\;\; 2 \;\; -1 \;\; -15 \;\; 0$

$(2x+5)(x-3) = 0$



$(-\infty, -\frac{5}{2}] \cup [1, 3]$

40) Solve the polynomial inequality:

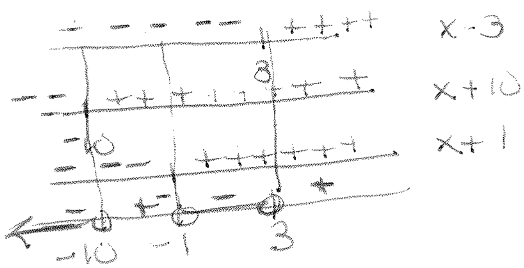
$(x + 10)(x + 1)(x - 3) < 0$

A) $(-10, -1) \cup (3, \infty)$

B) $(-\infty, -1)$

C) $(-\infty, -10) \cup (-1, 3)$

D) $(3, \infty)$



41. $f(x) = \dfrac{2x+6}{x^2+7x+12} + 3$

Factored Form: $f(x) = \dfrac{2(x+3)}{(x+3)(x+4)} + 3$

Holes: $(-3, 5)$

Slant Asymptote: none

Vertical Asymptote: $x = -4$

Horizontal Asymptote: $y = 3$

42. $f(x) = \dfrac{2x+4}{x^2-x-6} + 5$

Factored Form: $f(x) = \dfrac{2(x+2)}{(x-3)(x+2)} + 5$

Holes: $(-2, 4\frac{3}{5})$

Slant Asymptote: none

Vertical Asymptote: $x = 3$

Horizontal Asymptote: $y = 5$

43. $f(x) = \dfrac{x^2+2x+5}{x-4}$

$x-4 \,\overline{)\, x^2+2x+5}$ quotient $x+6$
$x^2 - 4x$
$6x+5$

Slant Asymptote: $y = x + 6$

Vertical Asymptote: $x = 4$

Horizontal Asymptote: none

44. $f(x) = \dfrac{5}{x^2 - 16} + 3x - 5$

Slant Asymptote: $y = 3x - 5$

Vertical Asymptote: $x = 4,\ x = -4$

Horizontal Asymptote: none

45. $f(x) = \frac{3}{x^2-4} -2x+7$

Slant Asymptote: y = -2x+7

Vertical Asymptote: x=2 or x=-2

Horizontal Asymptote: none

46. $f(x) = \frac{7}{x^2-25} + 7x+2$

Slant Asymptote: y=7x+2

Vertical Asymptote: x=5 or x=-5

Horizontal Asymptote: none

47. Given the following function: $f(x) = \frac{1}{x^3}$ has been graphed, what transformations would need to be completed in order to graph:

a) $g(x) = -f(x+4) - 6$

reflection over the x axis
Left 4
down 6

b) $g(x) = \frac{1}{5}f(x-3) + 2$

compressed by a factor of 1/5
Right 3
up 2

48. What are the zeros and asymptotes for the function below?

$$f(x) = \frac{x^2+7x+12}{x^2-3x+2} \quad \frac{(x+4)(x+3)}{(x-2)(x-1)}$$

VA: x=2, x=1

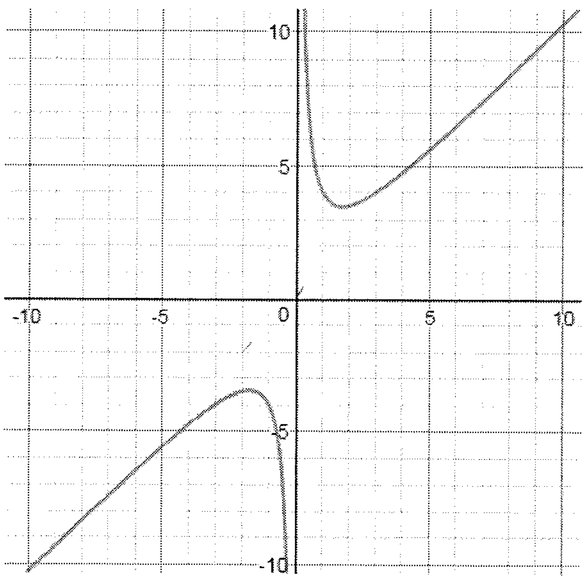
HA: y=1

zeros: {-4, -3}

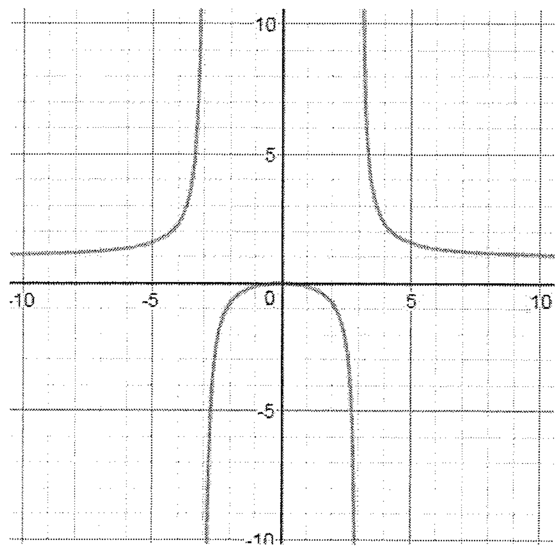
Match the function with the graph. Consider asymptotes and a table of points.

49.	50.	51.	52.
$f(x) = \dfrac{x^2}{x^2 - 9}$	$f(x) = \dfrac{x^2 - 5}{x}$	$f(x) = \dfrac{x^2 + 3}{x}$	$f(x) = \dfrac{x - 1}{x + 2}$
VA $x = \pm 3$ HA $y = 1$ B	VA $x = 0$ $f(1) = -4$ $x \overline{)x^2 - 5}$ with x^2 above and x^2 below SA $y = x$ D	VA $x = 0$ SA $y = x$ $f(1) = 4$ A	VA $= -2$ $y = 1$ $(0, -\frac{1}{2})$ C

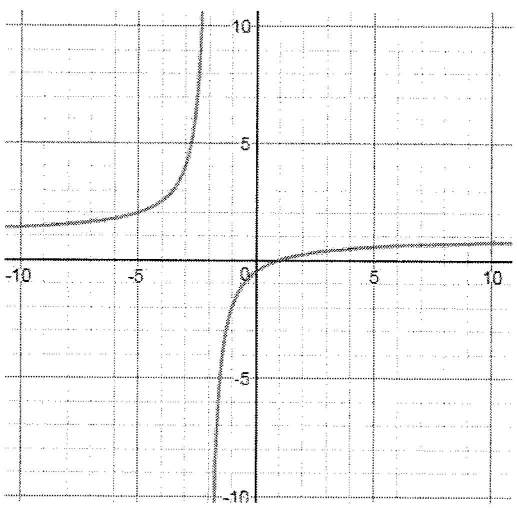
A.



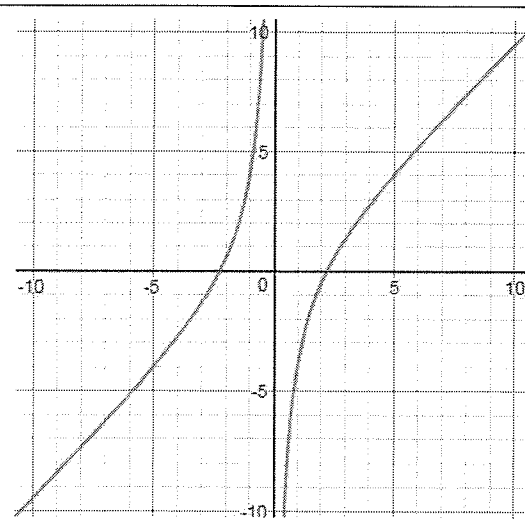
B.



C.



D.



53. Factor and Solve to find the zeroes

$$f(x) = 3x^4+28x^2+9$$

$= (3x^2+1)(x^2+9)$

$3x^2=-1$

$x^2=-1/3$

$x=\pm\sqrt{\frac{-1}{3}\cdot\frac{3}{3}}$

$x=\pm\frac{i\sqrt{3}}{3}, \pm 3i$

$\{\pm\frac{\sqrt{3}}{3}i, \pm 3i\}$

54. Factor and Solve to find the zeroes

$$f(x) = 3x^4-47x^2-16$$

$= (3x^2+1)(x^2-16)$

$= (3x^2+1)(x+4)(x-4)$

$x^2=-1/3$ $x^2=16$

$x=\pm 4$

$x=\frac{\pm i\sqrt{3}}{3}, \pm 4$

$\{\pm\frac{\sqrt{3}}{3}i, \pm 4\}$

55. ~~Factor and Solve to find the zeroes~~

~~$f(x) = 4x^4+101x^2+25$~~

Find the zeros, given 4 is a zero.

$f(x)=2x^3+3x^2-39x-20$

4	2	3	-39	-20
		8	44	20
	2	11	5	

$(2x+1)(x+5)$

$\{-5, -1/2, 4\}$

56. ~~Factor and Solve to find the zeroes~~

~~$f(x) = 2x^4+75x^2+37$~~

Find the zeros, given -5 is a zero.

$f(x)=4x^3+9x^2-52x+15$

-5	4	9	-52	15
		-20	55	-15
	4	-11	3	0

$4x^2-11x+3$

$\frac{11\pm\sqrt{121-4(4)(3)}}{8}$

$\frac{11\pm\sqrt{121-48}}{8}$

$x=\frac{11\pm\sqrt{73}}{8}$

$\{-5, \frac{11\pm\sqrt{73}}{8}\}$