

HONORS PRECALC
Rational Functions and Rational Inequalities Review

NAME Key
PERIOD _____

Complete each of the following:

To specify the domain of a rational function: *exclude any values that make the denominator equal zero.*

To find the x-intercept(s) of a rational function: *let $y=0$ and solve for x*

To find the y-intercept of a rational function: *let $x=0$ and solve for y*

To find any vertical asymptote(s) of a rational function: *Find values of x that make only the denominator equal zero*

To find any removable discontinuities (holes): *Find values of x that make both the numerator and denominator equal zero. To find the y values, cross out common factor and plug in for x to find y .*

To find the horizontal asymptote (if any) when the function is in the form $R(x) = \frac{P(x)}{Q(x)}$; $Q(x) \neq 0$? *Compare the degree of numerator to degree of denominator*

Degree in numerator is bigger: *NO HA*

Degree in denominator is bigger: *$y=0$*

Degrees are the same: *$y = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$*

When might there be a slant asymptote? How do you find it when the function is in the form $R(x) = \frac{P(x)}{Q(x)}$; $Q(x) \neq 0$? *if the degree of the numerator is exactly one more than the degree of the denominator. Do long division.*

If the rational function is in the form: $g(x) = \frac{A}{Bx+C} + D$, how do A, B, C and D affect the transformation of $f(x) = \frac{1}{x}$?

A: vertical stretch by factor of A

B: horizontal compression by factor of B

$\frac{C}{B}$: vertical translation

$x = \frac{C}{B}$
vertical asymptote

D: horizontal translation
 $y = D$ horizontal asymptote.

Identify each of the following for the given function. If it has none, write NONE

1. $f(x) = \frac{x-4}{2x^2-8x}$
 Domain: $\{x | x \neq 0, 4\}$
 x-intercept: none
 y-intercept: none
 hole(s): $(4, 1/8)$
 Vertical asymptote(s): $x=0$
 Horizontal asymptote: $y=0$
 Slant asymptote: none

2. $f(x) = \frac{6x^2+7x-3}{x^2+4}$
 Domain: $\{x | x \in \mathbb{R}\}$
 x-intercept: $(-3/2, 0), (1/3, 0)$
 y-intercept: $(0, -3/4)$
 hole(s): none
 Vertical asymptote(s): none
 Horizontal asymptote: $y=6$
 Slant asymptote: none

3. $f(x) = \frac{x^2+5}{x+1}$
 Domain: $\{x | x \neq -1\}$
 x-intercept: none
 y-intercept: $(0, 5)$
 hole(s): none
 Vertical asymptote(s): $x=-1$
 Horizontal asymptote: none
 Slant asymptote: $y=x-1$

4. $f(x) = \frac{2x^3-x^2-x}{x^2+2x+1}$
 Domain: $\{x | x \neq -1\}$
 x-intercept: $(-1/2, 0), (0, 0), (1, 0)$
 y-intercept: $(0, 0)$
 hole(s): none
 Vertical asymptote(s): $x=-1$
 Horizontal asymptote: none
 Slant asymptote: $y=2x-5$

6. $f(x) = \frac{x^2-4x-12}{x^2+5x+6}$
 Domain: $\{x | x \neq -3, -2\}$
 x-intercept: $(6, 0), (-2, 0)$
 y-intercept: $(0, -2)$
 hole(s): $(-2, -8)$
 Vertical asymptote(s): $x=-3$
 Horizontal asymptote: $y=1$
 Slant asymptote: none

7. $f(x) = \frac{3}{x-6} + 3$
 Domain: $\{x | x \neq 6\}$
 x-intercept: $(5, 0)$
 y-intercept: $(0, 5/2)$
 hole(s): none
 Vertical asymptote(s): $x=6$
 Horizontal asymptote: $y=3$
 Slant asymptote: none

8. $f(x) = \frac{2x-1}{x^2-49} + 3x - 2$
 Domain: $\{x | x \neq \pm 7\}$
 x-intercept: $(0.668, 0)$
 y-intercept: $(0, -1.49)$
 hole(s): none
 Vertical asymptote(s): $x=\pm 7$
 Horizontal asymptote: none
 Slant asymptote: $y=3x-2$

9. $f(x) = \frac{4x-16}{x^2-2x-8} + 3$
 Domain: $\{x | x \neq -2, 4\}$
 x-intercept: $(-3/2, 0)$
 y-intercept: $(0, 5)$
 hole(s): $(4, 3/3)$
 Vertical asymptote(s): $x=-2$
 Horizontal asymptote: $y=3$
 Slant asymptote: none

11. $f(x) = \frac{x^2+x-30}{x^2-8x+15}$
 Domain:
 x-intercept:
 y-intercept:
 hole(s):
 Vertical asymptote(s):
 Horizontal asymptote:
 Slant asymptote:

Answer the questions below:

12. Which function below has a slant asymptote at $y=2x-5$ and vertical asymptotes at $x=1$ and $x=-1$?

- A) $f(x) = \frac{x^2-1}{2x-5}$
 B) $f(x) = \frac{x-1}{x+1} + 2x - 5$
 C) $f(x) = \frac{-2}{x^2-1} + 2x - 5$
 D) $f(x) = \frac{2x-5}{x+1} + x - 1$

13. Describe the transformation of $f(x) = \frac{1}{x}$ for each of the following:

- A) $-f(x+3) - 2$
 B) $3f(x-1) + 2$

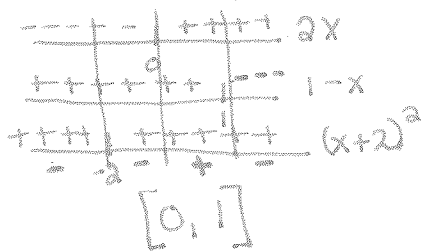
Handwritten notes for Q13:
 For A: $f(x) = \frac{1}{x}$
 $f(x+3) = \frac{1}{x+3}$
 $-f(x+3) = \frac{-1}{x+3} - 2$
 For B: $\frac{3}{x-1} + 2$
 Vertical stretch by factor of 3, right 1, up 2.

needed calc sorry! f not easiest like this

Solve the inequality:

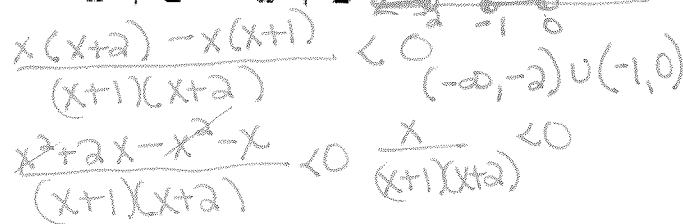
14.

$$\frac{2x(1-x)}{(x+2)^2} \geq 0.$$



15.

$$\frac{x}{x+1} < \frac{x}{x+2}$$



16.

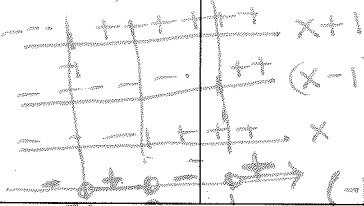
$$x(x-1) > \frac{x-1}{x}$$

$$x \frac{x(x-1) - x-1}{x} > 0$$

$$\frac{x^3 - x^2 - x + 1}{x} > 0$$

$$\frac{x^2(x-1) - 1(x-1)}{x} > 0$$

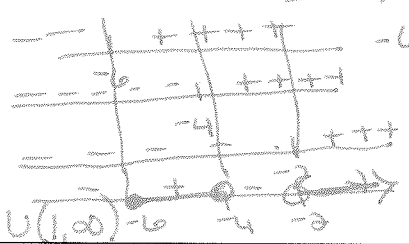
$$\frac{(x+1)(x-1)^2}{x} > 0$$



17.

$$\frac{x+6}{x^2+6x+8} \geq 0$$

$$\frac{x+6}{(x+4)(x+2)} \geq 0$$



Answer each of the following: (polynomial review)

18.

Use Descartes's rule of signs to determine the number of possible positive, negative, and imaginary zeros for:

$$f(x) = -3x^4 + 2x^3 + x^2 + 3x - 2 \quad 2, 0 \text{ pos}$$

$$f(-x) = -3x^4 - 2x^3 + 2x^2 - 3x - 2 \quad 2, 0$$

2 pos 2 neg 0 mag
2 pos 0 neg 2 imaginary
0 pos 2 neg 2 imaginary

19. Find all the zeros of:

$$f(x) = x^4 + x^3 - 12x^2$$

$$x^2(x^2 + x - 12)$$

$$x^2(x+4)(x-3)$$

0 mult is 2

$$\{-4, 0, 3\}$$

20. Find all the zeros of the function given 3 is a zero.

$$f(x) = x^3 - 3x^2 + 4x - 12$$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 4 & -12 \\ & & 3 & 0 & 12 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$$\{3, \pm 2i\}$$

$$x^2 + 4 = 0$$

$$x^2 = -4$$

$$x = \pm 2i$$

Match each rational function with its graph a-h.

21. $f(x) = -\frac{2}{x}$ c

22. $f(x) = \frac{e-1}{x-2}$ $y=0$ $x=a$

23. $f(x) = \frac{b}{x-2}$ $x=a$ $y=1$

24. $f(x) = \frac{x-2}{x}$ $y=1$ $x=0$

25. $f(x) = \frac{1}{x(x-2)}$ g $x=0$ $x=2$

26. $f(x) = \frac{h}{x^2-4}$ $y=1$ $x=\pm 2$

27. $f(x) = \frac{-x+4}{x^2-4}$ f $y = -\frac{1}{2}x + 2 = a$

28. $f(x) = \frac{x^2+2x+1}{x}$ $(x+1)(x+1)$ x

