Solving Rational Inequalities

PART 1

In order to solve a rational inequality, it MUST have ZERO on one side and be simplified to a SINGLE RATIONAL EXPRESSION on the other.

Example:

$$\frac{x^2-9}{x^2-1} < 0$$

$$\frac{x^2 - 9}{x^2 - 1} < 0$$

Now follow these steps…

Step 1: Factor the numerator and denominator of the rational expression and find all the values of x that make the numerator=0 and the denominator=0.

(numerator) (x + 3) (x – 3) so… x = 3 and -3

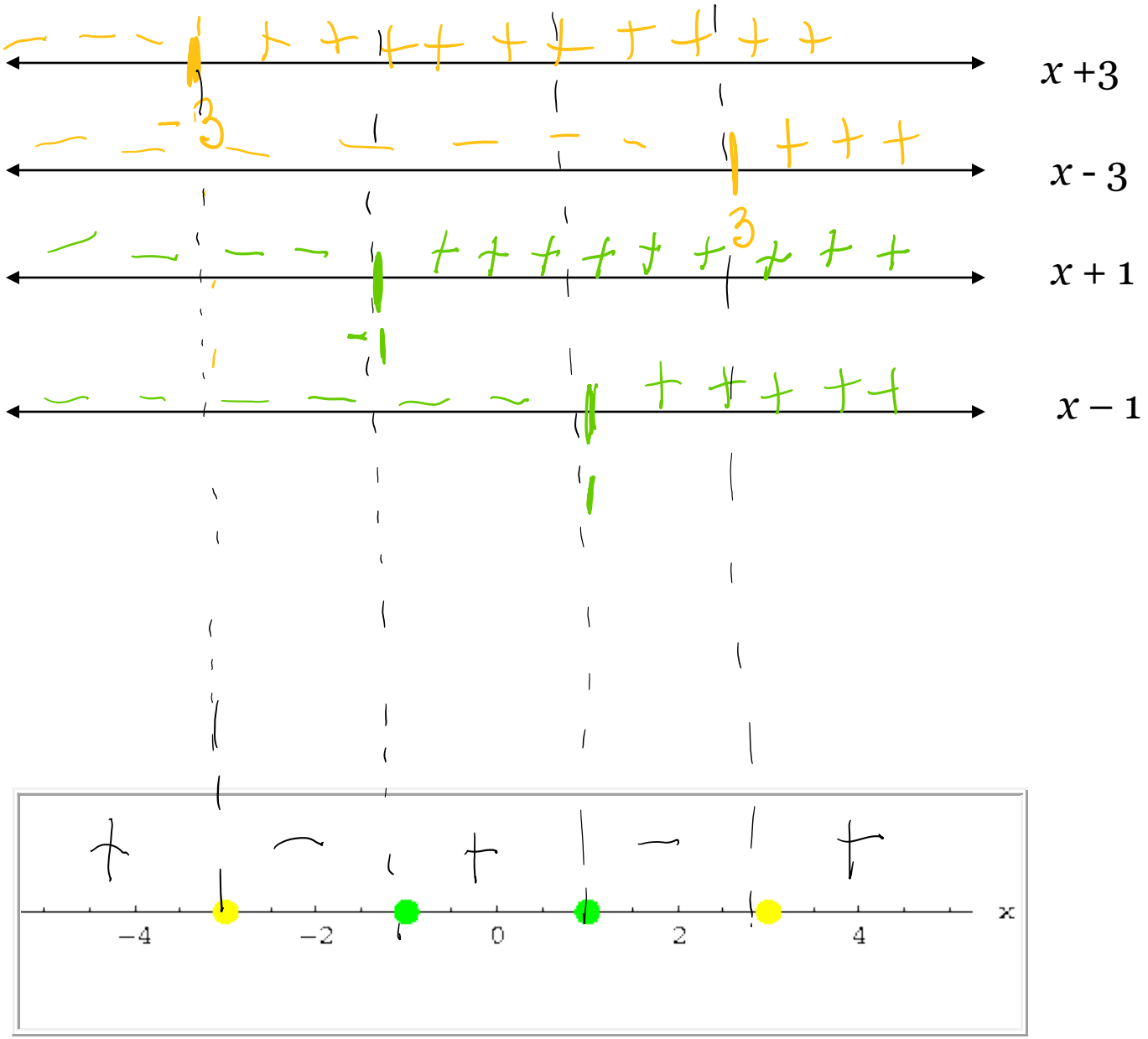
(denominator) (x + 1) (x – 1) so… x = 1 and -1

Step 2: Draw a number line for each factor and mark where the factor is positive/negative.

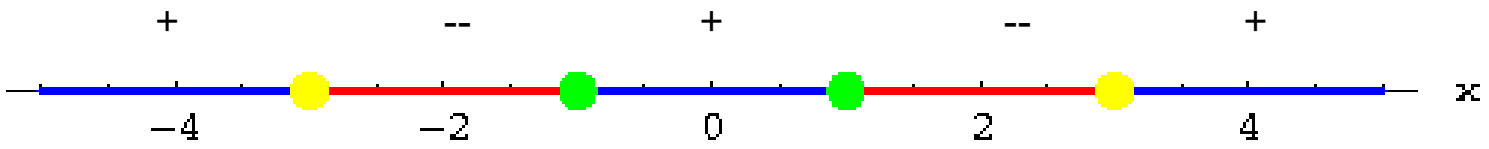
Step 3: Draw a picture of the x-axis and mark these points. These are often called "critical points".

Step 4: Determine if the ratio is positive/negative in each interval formed by the critical points.

Step 5: Determine the solution based on the inequality.


x +3
x - 3
x + 1
x − 1
−4
−2
0
2
4
x

We now have:



SOLUTION: Since we want the expression to be LESS THAN 0, so we are looking for the intervals that are negative.

The solution is .... **(-3, -1) ∪ (1, 3).**

NOTE: If an inequality has ≥ or ≤ then you would use brackets [ ] around the intervals, unless the value makes the denominator undefined.

$$\frac{x-1}{x+2} \leq 0$$

(-2, 1]

$$\frac{x^2+5x+6}{x^2-4x-5} \geq 0$$

(-∞, -3] ∪ [-2,-1) ∪(5, ∞)

$$\frac{x^2-16}{x^2-3x+2} < 0$$

(-4,1) ∪ (2,4)

Solving Rational Inequalities

PART 2

WHAT DO I DO WHEN THE PROBLEM ISN'T A SINGLE RATIONAL EXPRESSION ON ONE SIDE?

Solving Rational Inequalities 2

Remember that you must COMBINE all fractions together on ONE SIDE so it is set to 0.

$$\frac{2}{x-4} + \frac{1}{x+1} > 0$$

How would we combine this into a single fraction on the left?

Solving Rational Inequalities 2

$$\frac{2(x+1)}{(x+1)(x-4)} + \frac{1(x-4)}{(x+1)(x-4)} \geq 0$$

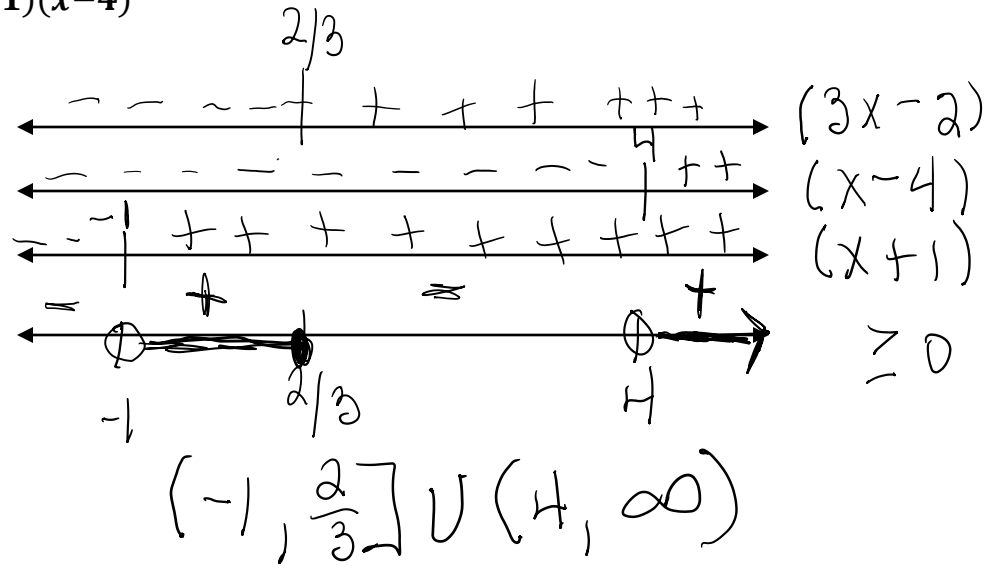
Get a common denominator

$$\frac{2x+2+x-4}{(x+1)(x-4)} \geq 0$$

Add

$$\frac{3x-2}{(x+1)(x-4)} \geq 0$$

Simplify



Solve

$$(-1, \tfrac{2}{3}] \cup (4, \infty)$$

Try this one:

$$\frac{1}{x+2}+\frac{3}{x-4}<0$$

What if there are terms on BOTH sides?

$$\frac{2x}{4} - \frac{5x+1}{3} > 3$$

$$\frac{x^2-4}{x^2-1} \leq \frac{x}{x+3}.$$

Additional Practice:

1) $\frac{4}{x-6}+\frac{2}{x+1}>0$

2) $\frac{x+6}{4x-3}\geq 1$

3) $\frac{x^2-x-11}{x-2}\leq 3$

4) $\frac{1}{x}>\frac{1}{x+5}$

5) $\frac{x}{x+1}-\frac{x-1}{x}<\frac{1}{20}$