Graphs of Rational Functions

- Rational Function in the form: $f(x) = \frac{A}{Bx-C} + D$

- A is the vertical stretch
- B is the horizontal compression
- $\frac{C}{B}$ is the horizontal translation

  $x = \frac{C}{B}$ is the vertical asymptote
- D is the vertical translation

  $y =$ D is the horizontal asymptote

EXAMPLE

- How can the graph of $g(x) = \frac{3}{4x-20} + 1$ be obtained from the graph of the parent function $f(x) = \frac{1}{x}$?
- What are the horizontal and vertical asymptotes of g(x)?

- Stretch vertically by a factor of 3, compress horizontally by a factor of 4, shift right 5 units, shift up 1 unit.
- HA: $y = 1$
- VA: $x = 5$

Example

- Write the function $g(x)$ that transforms the function $f(x) = \frac{1}{x}$ as follows:
  3 units right
  stretched vertically by a factor of 0.5
  compressed horizontally by a factor of 3

- A = .5
  B = 3
  Since it is moved three to the right, $\frac{C}{B}$=3 and since B is 3, C has to = 9
  Therefore:

$$g(x) = \frac{.5}{3x - 9}$$

More Rational Functions

What about rational functions that are not in the form given on the previous slides?

$R(x) = \frac{P(x)}{Q(x)}$; where $P(x)$and $Q(x)$are polynomial functions and $Q(x) \neq 0$

Graphical Features of R(X)=$\frac{P(x)}{Q(x)}$

INTERCEPTS

x-intercept(s): let *y* = 0 and solve for *x*
y-intercept: let *x* = 0 and solve for *y*

EXAMPLE: $f(x) = \frac{x^2+5x+4}{2}$

x-intercept(s): (-4, 0), (-1, 0)

y-intercept: (0, 2)

Graphical Features of $R(x) = \frac{P(x)}{Q(x)}$

VERTICAL ASYMPTOTE(s) / HOLE(s)

Determine any value(s) that makes the denominator equal zero.

If that value makes ONLY the denominator 0, there is a vertical asymptote at that value.

If that value ALSO makes the numerator 0 there is a hole in the graph at that value.

EXAMPLE:

$f(x) = \frac{x+4}{x^2+5x+4} = \frac{x+4}{(x+4)(x+1)}$ Has a VA at $x = -1$

and a hole at $(-4, -\frac{1}{3})$

Graphical Features of $R(x) = \frac{P(x)}{Q(x)}$

HORIZONTAL ASYMPTOTE

Determine the degree of the numerator and the degree of the denominator.

If the degree of the denominator is greater than the degree of the numerator, there is a HA at $y = 0$

If the degree of the denominator is the same as the degree of the numerator there is a HA

at $y = \frac{Leading\ coefficient\ of\ numerator}{Leading\ coefficient\ of\ denominator}$

If the degree of the numerator is greater than the degree of the denominator there is no HA….HOWEVER

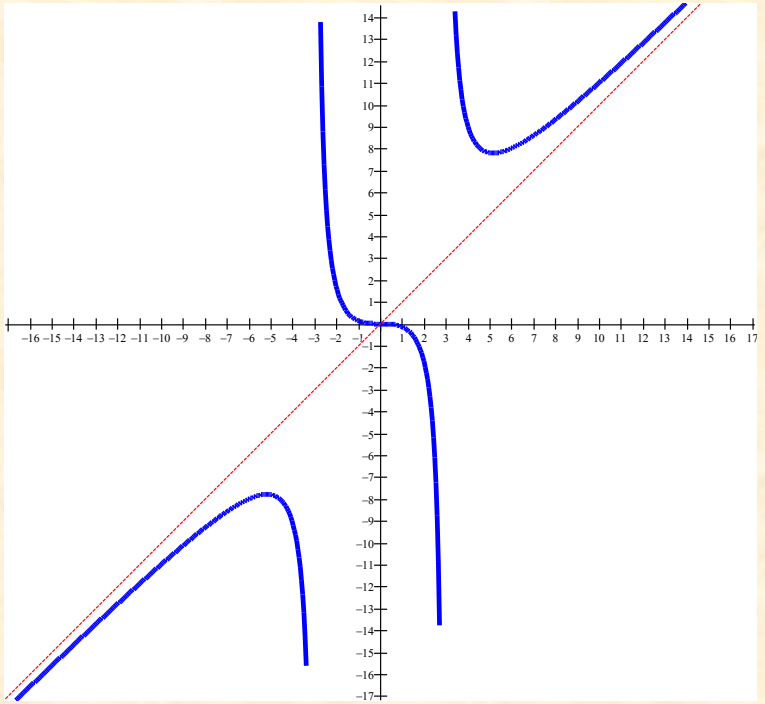
Graphical Features of Rational Functions

…If the degree of the numerator is EXACTLY one more than the degree of the denominator:

Slant (or Oblique) Asymptotes: occur if the degree of numerator is exactly one more than the degree of denominator. Use polynomial long division, the quotient is the equation for the slant asymptote.

Note: Graphs **NEVER** intersect their vertical asymptotes but they can intersect slant and horizontal asymptotes.

NOTE: graph never has both a HA and SA



Example 1

List the asymptotes and intercepts for the following graph.

$$f(x) = \frac{4}{x^2 + 1}$$

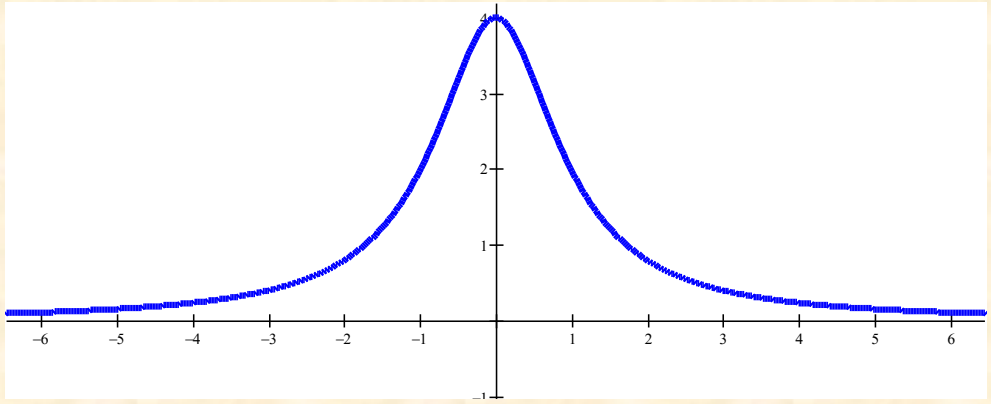
Vertical asymptote: none

Horizontal asymptote: y=0 $\frac{\text{small}}{\text{large}}$

Slant asymptote: none

x-intercept: none

y-intercept: (0,4)



Example 2

List the asymptotes and intercepts for the following graph.

$$f(x) = \frac{x^2 - x}{x + 1}$$

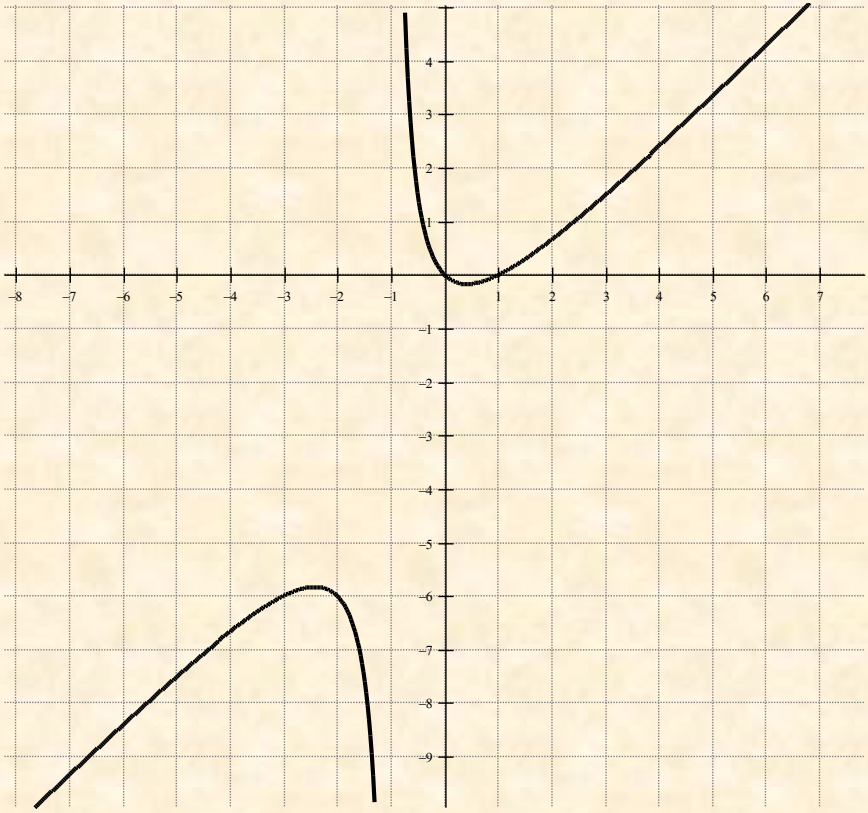
Vertical asymptote: x = –1

Horizontal asymptote: none $\frac{\text{large}}{\text{small}}$

Slant asymptote: y = x – 2

x-intercept: (0,0),(1,0)

y-intercept: (0,0)



Example 3

List the asymptotes and intercepts for the following graph.

$$f(x) = \frac{3x^2}{x^2 - 4}$$

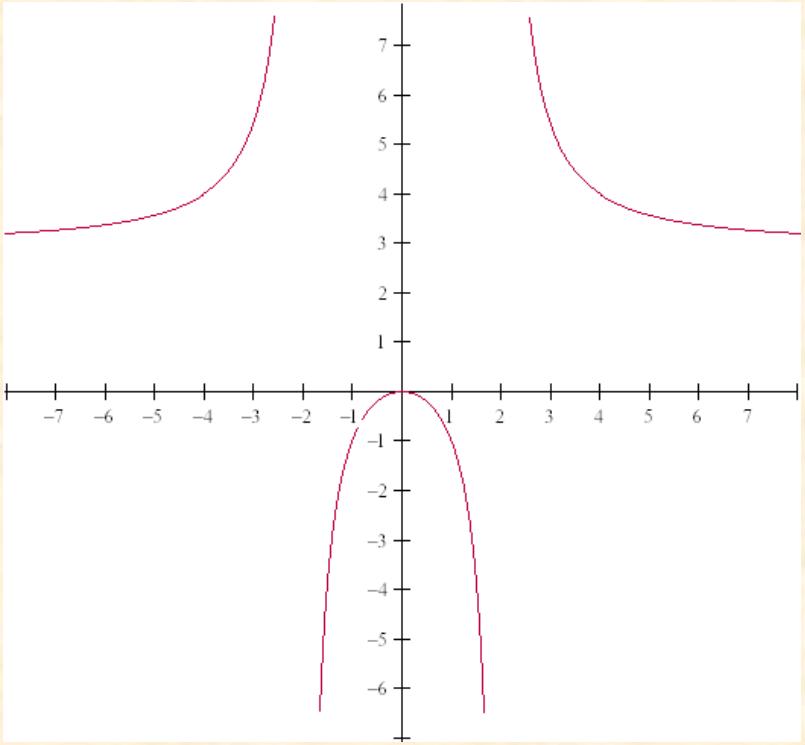
Vertical asymptote: x = –2 , 2

Horizontal asymptote: y=3 $\frac{same}{same}$

Slant asymptote: none

x-intercept: (0,0)

y-intercept: (0,0)



Example

- Consider: $f(x) = \frac{x^2+4x-1}{x+3}$

- There is a VA at x = -3
  Find the slant asymptote:

  This function can also be written:
  $f(x) = \frac{-4}{x+2} + x + 1$ (why)
  Observe: what is the VA and slant asymptote?

Example

- Consider: $f(x) = \frac{4}{x^2-64} - 3x + 2$

What are the asymptotes of this equation?